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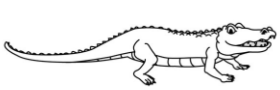
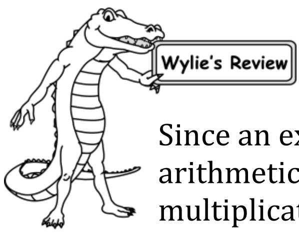


Table of Contents with Standards

Multiply Two Linear Expressions

Generate equivalent algebraic expressions.

Apply properties of operations to multiply two linear expressions with rational coefficients.



Since an expression represents a calculation with numbers, we use the same rules of arithmetic. Recall the reordering and regrouping rules for addition and multiplication respectively:

$$a + b = b + a \qquad \text{and} \qquad ab = ba$$

$$a + (b + c) = (a + b) + c \qquad \text{and} \qquad a(bc) = (ab)c$$

We can also write subtraction as addition:

$$a - b = a + (-b)$$

The **distributive property** shows how multiplication by the number a distributes over the sum $b + c$:

$$a(b + c) = ab + ac.$$

Example 1: Simplify the following expressions The solutions are provided with problem statement:

(a) $4(2x + 5) = 4 \cdot 2x + 4 \cdot 5 = 8x + 20$

(b) $-2.1(4a + 3b - 6) = (-2.1) \cdot 4a + (-2.1) \cdot 3b - (-2.1) \cdot 6 = -8.4a - 6.3b + 12.6$

(c) $(1.6x - 2.8) \cdot (0.9x) = (1.6x) \cdot (0.9x) - (2.8) \cdot (0.9x) = 1.44x^2 - 2.52x$

One purpose for reordering terms in an expression is to put like terms next to each other so that they can be combined.

Example 2: Simplify the following expressions.

(a) $3(x + 3) - x - 12$ (b) $\frac{x}{4}(8x - 4) + x^2$

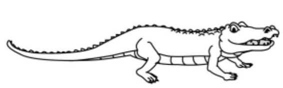
Solution:

(a) We first distribute 3: $3(x + 3) = 3x + 3 \cdot 3 = 3x + 9$. Then, by reordering and combining like terms, we obtain

$$3(x + 3) - x - 12 = 3x + 9 - x - 14 = 3x - x + 9 - 12 = 2x - 3.$$

(b) We begin by multiplying $\frac{x}{4}$ by $(8x - 4)$ and then combining similar terms:

$$\frac{x}{4}(8x - 4) + x^2 = \frac{x}{4} \cdot 8x - \frac{x}{4} \cdot 4 + x^2 = \frac{8}{4} \cdot x \cdot x - \frac{4}{4}x + x^2 = 2x^2 - x + x^2 = 3x^2 - x.$$



Now Try These:

For 1-7, Hot Text: Are the expressions equivalent?

YES	NO

1. $5 + \frac{x}{2}$ and $5 + 0.5x$

2. $3(z + w)$ and $3z + 3w$

3. $-a + 2$ and $-(a + 2)$

4. $bc - cd$ and $c(b - d)$

5. $(a - b)^2$ and $a^2 - b^2$

6. $3h^2 + 2h^2$ and $5h^2$

7. $3b + 2b^2$ and $5b^3$

For 8-13, Equation Editor: Simplify each expression.

8. $-0.5(8.6x - 1.2)$

9. $(-1.3)^2(1 + x)$

10. $3.8(-2x + 5)$

11. $(3x + 7.5)(1.2x)$

12. $(-4.9x)(7.3 - 3.8x)$

13. $(-100)(2.5x + \frac{1}{5})$

14. Multiselect: Which of the following are equivalent to $4x(\frac{1}{2}x - 2)$? Choose all that apply.

A. $2x - 2$

B. $2x^2 - 8x$

C. $2(x^2 - 4x)$

D. $2x^2 - 8$

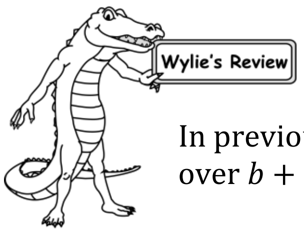
E. $x(2x - 8)$

F. $4x - 4$

Rewrite Expressions with Common Monomial Factor

Generate equivalent algebraic expressions.

Rewrite the sum of two algebraic expressions having a common monomial factor as a common factor multiplied by the sum of two algebraic expressions.



In previous section we learned the **distributive property** where a is distributed over $b + c$:

$$a(b + c) = ab + ac.$$

Other times, we use the distributive property in the reverse way: $ab + ac \quad \rightarrow \quad a(b + c)$.

We call this taking out a common factor a from $ab + ac$.

Example 1: Take out a common factor. The solutions are provided with the problem statement.

(a) $2x + xy = x(2 + y)$

(b) $-wx - 1.2w = -w(x - 1.2)$

(c) $12nm^2 - m^2 = m^2(12n - 1)$

(d) $-4xyz - 8xy = -4xy(z + 2)$

(e) $-ab + a^2b - ab^2 = ab(-1 + a - b) = -ab(1 - a + b)$

Example 2: Mrs. Navetta asked Carla to write the following solutions on the board. How many errors did Carla make? Can you fix her mistakes?

Carla's work:

a) $2x + 3y = x(2 + 3y)$

b) $3x + 6xy = 3x(1 + 2y)$

c) $20xy + 25xm = 15x(2y + 3m)$

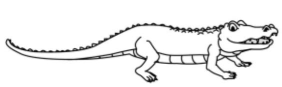
d) $-5xy + 25ax = 5x(5a - y)$

Solution:

Carla made a mistake in problems a) and c).

In problem a), these two terms have no common factors, so there is nothing to factor out. The problem has to remain $2x + 3y$.

In problem c), the greatest common factor for 20 and 25 is 5 (not 15). We can therefore factor out a 5. We can also factor out x as it is common for both terms. Thus, the correct solution is $20xy + 25xm = 5x(4y + 5m)$.



Now Try These:

For 1-7, Hot Text: Are the two expressions equivalent?

YES NO

1. $10 + \frac{x}{2}$ and $\frac{1}{2}(20 + x)$

2. $5x + 100$ and $5(x + 500)$

3. $3x^2 + 6x + 3$ and $3(x^2 + 2x)$

4. $a^2b + ab^2$ and $a(a + b)$

5. $a^2b + ab^2 + ab$ and $ab(a + b + 1)$

6. $6x + 8y$ and $2(3x + 4y)$

7. $3b + 2b^2$ and $b(3 + b)$

For 8-17, Equation Editor: Rewrite each expression by taking out the common factors.

8. $2ax - 3bx$

9. $5x + 100$

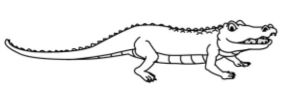
10. $22x^2 - 11$

11. $2x^2 - 6x$

12. $\frac{x}{5} + \frac{y}{5}$

13. $-m^2n - 3mn^2$

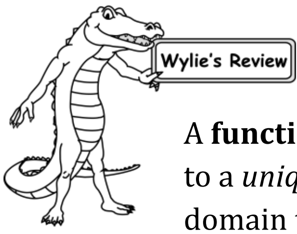
14. $-4a^2b - 6ab^2 - 2ab$



Determine whether the Relationship is a Function

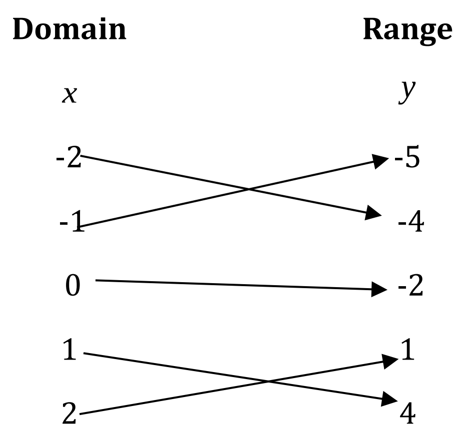
Define, evaluate, and compare functions.

Given a set of ordered pairs, a table, a graph or mapping diagram, determine whether the relationship is a function. Identify the domain and range of the relation.

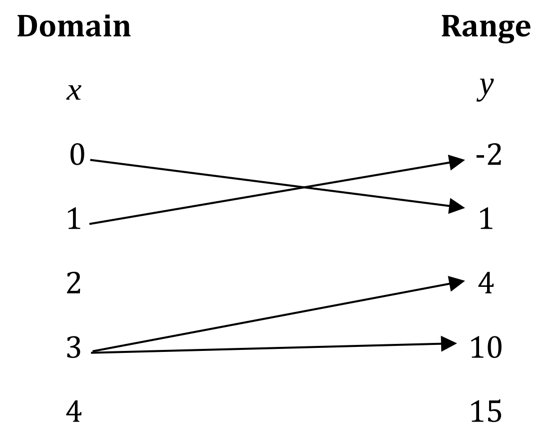


A **function** is a mapping that takes elements of one set, the **domain**, and maps them to a *unique* element in another set, the **range**. That is, for every element in the domain there is EXACTLY ONE element in the range.

The following example is a function because each element in the domain has one arrow pointing to a single element in the range.



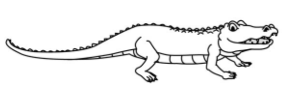
The following example is NOT a function because there are two arrows from 4 in the domain pointing to different elements in the range.



As an example, consider the following function called *The Mailbox Function*. The domain of the Mailbox Function is the set of all letters to be delivered by Mailman Fritz. Fritz's job is to place each letter in the proper mailbox. The set of mailboxes is the range. Note that each letter corresponds to **EXACTLY** one mailbox. Each mailbox may receive any number of letters, 0, 1, 30 etc.

The **domain** of a function is the set of objects that can be inputted into the function. Points that make the denominator 0 or cause a negative number under a square root are not in the domain.

The **range** of a function is the set of objects that are outputted from the function, or those things to which items in the domain are mapped.



The variable that is inputted into the function is called the **independent variable**. The number that is the corresponding output is called the **dependent variable.** If $y = f(x)$, then x is independent and y is dependent.

The **graph of a function** is a graph of the equation $y = f(x)$. Given a graph of a function, the domain of the function is the set of x values that have a point on the graph. **NOTE:** If $y = f(x)$ is a function, then for every x value there must be ONLY one y value.

Example 1: Find the domain and range of the function.

x	-2	-1	0	1	2	3	4
y	-8	-4	0	-4	-8	-12	-16

The domain is the set of x values. In this case the domain is {-2, -1, 0, 1, 2, 3, 4}.

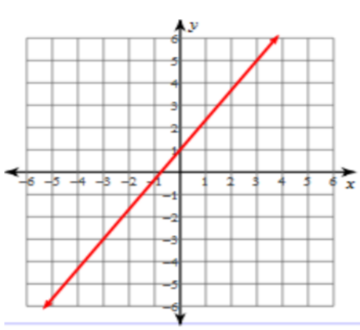
The range is the set of y values. -8 and -4 appear twice but we only list them once.

The range is $\{-16, -12, -8, -4, 0\}$.

Example 2: Find the domain of $f(x) = 2x - 1$.

Solution: If you take any real number, you can multiply it by 2 and still have a real number. Then if you subtract 1 you still get a real number. So in this case, the domain of $f(x) = 2x - 1$ is all real numbers or $(-\infty, \infty)$.

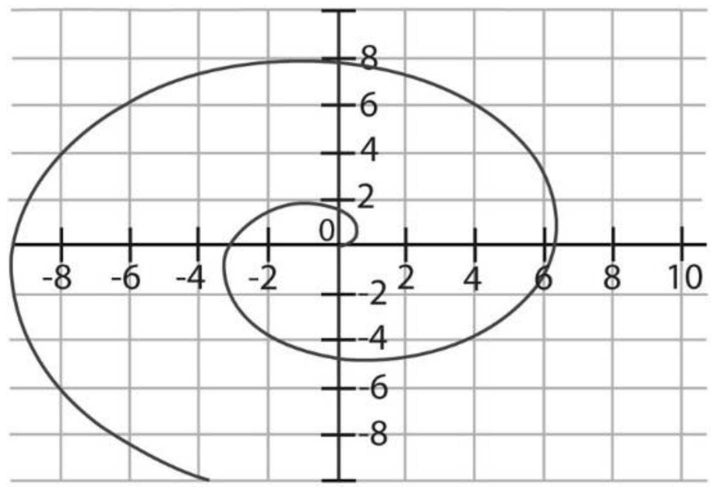
Example 3: The graph below is the graph of $f(x) = \frac{4}{3}x + 1$. Find the domain and range of $f(x)$.



Solution: The domain is the set of x-values that have corresponding points on the graph. In this case, the domain is all real numbers or $(-\infty, \infty)$.

The range is the set of y-values that have corresponding points on the graph. In this case, the range is all real numbers or $(-\infty, \infty)$.

Example 4: Is the following a graph of a function $y = f(x)$?



Solution: No. The graph is NOT the graph of a function because for example, when $x = 3$, there are at least 2 corresponding y values. To be a function, for each x value, there must correspond only one y value. Graphically this means that for every vertical line, it must intersect the graph AT MOST ONCE. It can intersect 0 times or 1 time, but no other intersection is allowed.

Vertical Line Test: For a graph, in the *xy*-plane, the graph represents a function if and only if every vertical line intersects the curve AT MOST ONCE.

Example 5: Determine if the relation is a function:

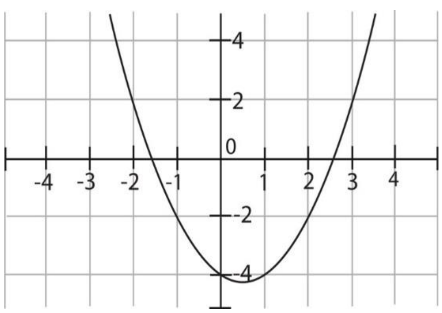
Solution: No. The graph is NOT a function because when x or the input is -7, there are two corresponding outputs or y values: 4 and 9.

Now Try These:

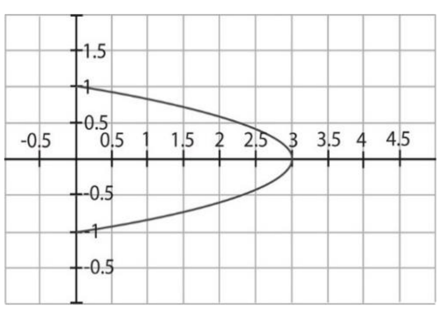
1. **Multiselect:** Which of the following sets of ordered pairs represents a function?
   A. $(-1,2), (3,4), (7,8), (-1,0)$
   B. $(1,2), (1,3), (3,4), (5,6)$
   C. $(1,2), (2,2), (3,2), (4,2)$
   D. $(-1,-1), (2,-1), (-1,2), (2,3)$
   E. (0, 4), (4, 0), (0, 7), (7, 0)
   F. (5, 3), (5, -3), (6, -3), (7, -3)

2. **Multiselect:** Which of the following are graphs of functions $y = f(x)$?

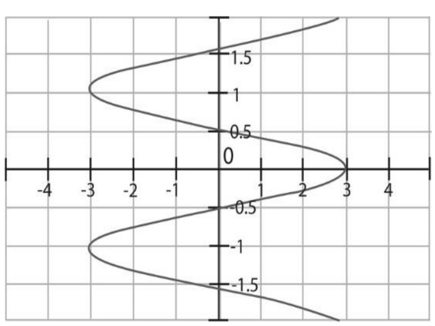
A.



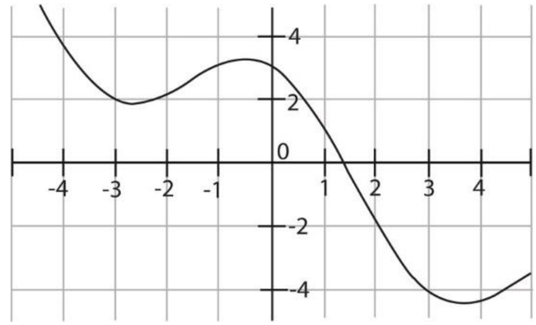
B.



C.



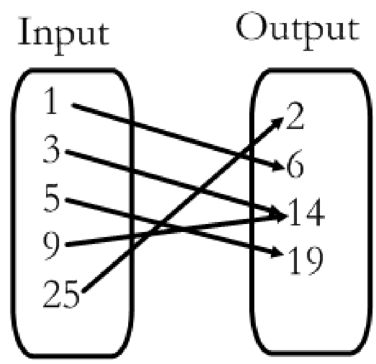
D.



E. All of them are functions.

F. None of them are functions.

3. Determine if the following relation is a function. Explain.



4. **Multiple Choice:** Given the following table, choose the correct response.

x	-2	-5	0	7
y	3	6	3	6

A. The relation is not a function because 3 is listed twice.

B. The relation is not a function because 6 is listed twice.

C. The relation is a function because for every x value, there is only one y value.

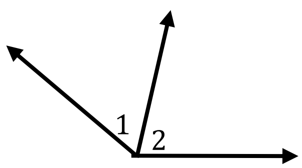
D. It cannot be determined whether or not this is a function.

Supplementary, Complementary, Vertical, and Adjacent Angles

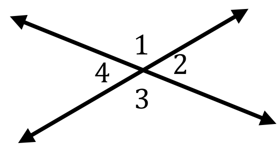
Develop an understanding of the Pythagorean Theorem and angle relationships involving triangles.

Solve mathematical problems involving the relationships between supplementary, complementary, vertical or adjacent angles.

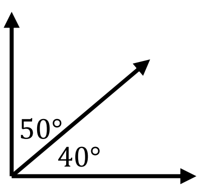
- **Adjacent angles** are two angles with a common side, a common vertex and no common interior points.



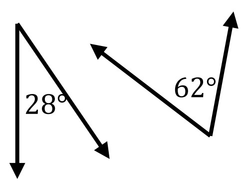
Adjacent angles:
$\angle 1$ and $\angle 2$
$\angle 2$ and $\angle 3$
$\angle 3$ and $\angle 4$
$\angle 1$ and $\angle 4$



- **Complementary angles** are two angles whose sum is 90°. Complementary angles may, or may not, be adjacent.

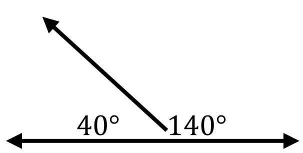


$50° + 40° = 90°$

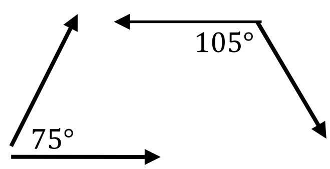


$28° + 62° = 90°$

- **Supplementary angles** are two angles whose sum is 180°. Supplementary angles may, or may not, be adjacent.

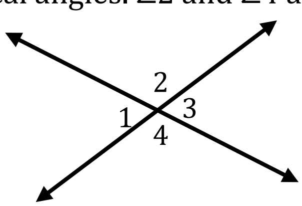


$40° + 140° = 180°$

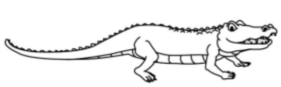


$75° + 105° = 180°$

- **Vertical angles** are opposite angles formed when two lines intersect. $\angle 1$ and $\angle 3$ are vertical angles. $\angle 2$ and $\angle 4$ are vertical angles.

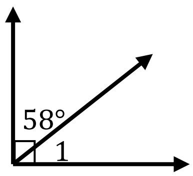


Vertical angles are congruent.
$\angle 1 \cong \angle 3$
$\angle 2 \cong \angle 4$

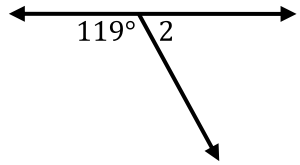


Example 1: Find the measure of each numbered angle.

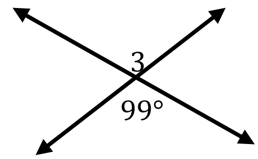
(a)



(b)



(c)



Solution:

(a) The two angles are complementary. They have a sum of 90°.

Complementary angles	$m\angle 1 + 58° = 90°$
Subtract 58°	$\underline{\quad -58° \quad -58°}$
Solve	$m\angle 1 = 32°$

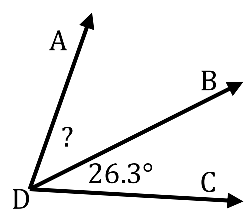
(b) The two angles are supplementary. They have a sum of 180°.

Supplementary angles	$m\angle 2 + 119° = 180°$
Subtract 119°	$\underline{\quad -119° \quad -119°}$
Solve	$m\angle 2 = 61°$

(c) The two angles are vertical. Vertical angles are congruent. Congruent angles have equal measures.

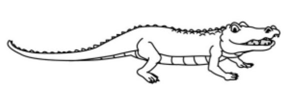
Vertical angles	$m\angle 3 = 99°$

Example 2: If the $m\angle ADC = 72.4°$, what is the measure of $\angle ADB$?

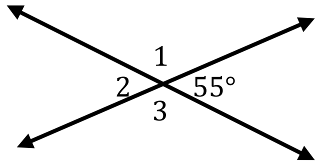


Solution:

The whole is equal to the sum of its parts	$m\angle ADB + m\angle BDC = m\angle ADC$
Substitution	$m\angle ADB + 26.3° = 72.4°$
Subtract 26.3°	$\underline{\quad - 26.3° \quad - 26.3°}$
Solve	$m\angle ADB = 46.1°$



Example 3: Find the measure of each numbered angle.



Solution:

The 55° angle and ∠1 are supplementary angles. Supplementary angles have a sum of 180°.

Equation	$55° + m\angle 1 = 180°$
Subtract 55°	$\underline{-55° \qquad\quad -55°}$
Solution	$m\angle 1 = 125°$

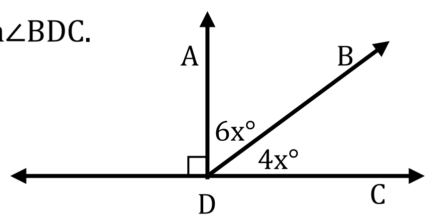
∠1 and ∠3 are vertical angles. Vertical angles are congruent.

Congruent angles have equal measures	$m\angle 1 = m\angle 3$
Substitution	$125° = m\angle 3$

The 55° angle and ∠2 are vertical angles. Vertical angles are congruent.
Therefore, m∠2 is 55°.

$m\angle 1 = 125°$, $m\angle 2 = 55°$, $m\angle 3 = 125°$.

Example 4: Find the value of x and m∠BDC.



Solution:

∠ADB and ∠BDC are complementary angles.

Complementary angles have a sum of 90°	$m\angle ADB + m\angle BDC = 90°$
Substitution	$6x + 4x = 90°$
Combine like terms	$10x = 90°$
Divide	$\frac{10x}{10} = \frac{90°}{10}$
Solve	$x = 9°$

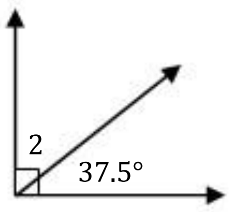
Find the m∠BDC	$m\angle BDC = 4x°$
Substitution	$m\angle BDC = 4(9°)$
Multiply	$m\angle BDC = 36°$

$x = 9°$ and $m\angle BDC = 36°$

Now Try These

Open Response: For 1-4**,** write an equation to find the measure of the numbered angle. Solve the equation.

1.



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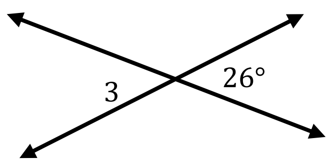

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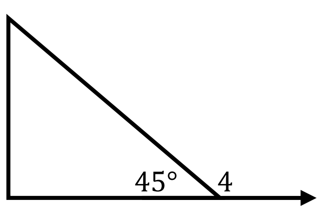

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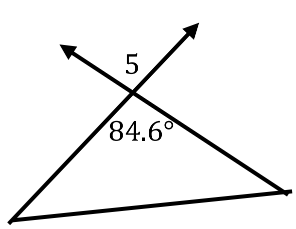
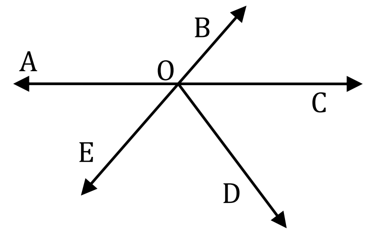
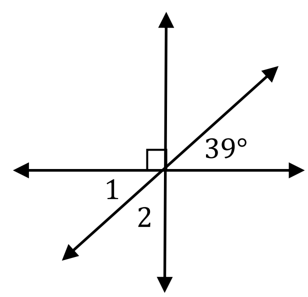


5. **Equation Editor:** Name an angle adjacent and supplementary to < AOB.



For 6-7: Use the diagram below to find the measures of the indicated angles.



6. **Multiple Choice**: What is the measure of $\angle 1$?

   A. 39°

   B. 90°

   C. 51°

   D. 30°

7. **Multiple Choice**: What is the measure of $\angle 2$?

   A. 39°

   B. 90°

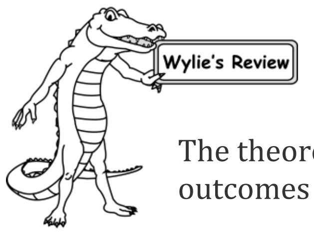
   C. 51°

   D. 30°

Find the Probability of an Event

Represent and find probabilities of repeated experiments.

Find the theoretical probability of an event related to a repeated experiment.



The theoretical probability is defined as the ratio of the number of favorable outcomes to the number of possible outcomes.

$$Probability\ of\ an\ event\ (E) = \frac{number\ of\ favorable\ outcomes}{number\ of\ possible\ outcomes}$$

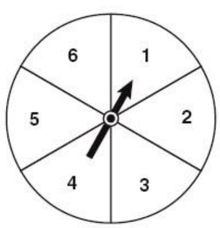
or

$$Probability\ of\ an\ event\ (E) = \frac{number\ of\ event\ outcomes}{number\ of\ outcomes\ in\ the\ sample\ space}$$

Determine the probability of rolling a 2 when a six-sided fair die is tossed.

We know that there are six possible outcomes: 1, 2, 3, 4, 5, 6. One of those outcomes is a 2. Therefore the probability of rolling a 2 is $\frac{1}{6}$.

Example 1: Mikeala spins the spinner below. Find the probability of each event.



(a) P(spinning a 5)

(b) P(not spinning a 6)

(c) P(spinning a number < 3)

(d) P(spinning a number > 2)

(e) P(spinning a number < 7)

(f) P(spinning a 7)

Solution:

(a) $\frac{1}{6}$

(b) $\frac{5}{6}$

(c) P(spinning 1 or 2) = $\frac{2}{6} = \frac{1}{3}$

(d) P(spinning 3, 4, 5, 6) = $\frac{4}{6} = \frac{2}{3}$

(e) $\frac{6}{6} = 1$

(f) $\frac{0}{6} = 0$

In Example 1e, it is guaranteed that the numbers 1, 2, 3, 4, 5, or 6 have a chance to be spun. There are 6 favorable outcomes out of 6 total outcomes. The probability of a guaranteed, or certain, event is 100%.

In Example 1f, it was not possible to spin the number 7. This event was impossible. There are 0 favorable outcomes out of 6 total outcomes. The probability of an event that is impossible equals 0

Example 2: Given a standard deck of 52 playing cards, determine the following probabilities. Write your answer as a percent. Round to the tenth place.

(a) P(heart)

(b) P(face card)

(c) P(6)

(d) P(10 of hearts)

Black Cards		Red Cards		
♣ Clubs	♠ Spades	♦ Diamonds	♥ Hearts	
Ace	Ace	Ace	Ace	
2	2	2	2	
3	3	3	3	
4	4	4	4	
5	5	5	5	
6	6	6	6	
7	7	7	7	
8	8	8	8	
9	9	9	9	
10	10	10	10	
Jack	Jack	Jack	Jack	Face cards
Queen	Queen	Queen	Queen	
King	King	King	King	

Solution:

(a) P(heart) $=\frac{13}{52} = 25\%$

(b) P(face card)=P(J,Q,K)= $\frac{12}{52} = 23.1\%$

(c) P(6)= $\frac{4}{52} = 7.7\%$

(d) P(10 of hearts)= $\frac{1}{52} = 1.9\%$

Example 3: Antonio has three marbles, two green (g) and one yellow (y). If Antonio replaces the first marble before picking the second one, determine the following probabilities. Express your answer as a decimal, rounded to the hundredth place.

(a) P(one green and one yellow)

(b) P(the same color twice)

(c) P(not picking green either time)

(d) P(both are green)

Solution:

You can use a two-way table and the sample space to help determine the probabilities:

Marbles	**g_1**	**g_2**	**y**
g_1	g_1g_1	g_1g_2	g_1y
g_2	g_2g_1	g_2g_2	g_2y
y	yg_1	yg_2	yy,

In a list format, the sample space is: g_1g_1, g_1g_2, g_1y, g_2g_1, g_2g_2, g_2y, yg_1, yg_2, yy.

(a) P(one green and one yellow) = $\frac{4}{9}$ = 0.44

(b) P(the same color twice) = $\frac{5}{9}$ = 0.56

(c) P(not picking green either time)= $\frac{1}{9}$= 0.11

(d) P(both are green) =$\frac{4}{9}$= 0.44

Example 4: Randall rolled a six-sided fair die two times. Determine the following probabilities. Express your answer as a simplified fraction.

(a) P(sum of 7)

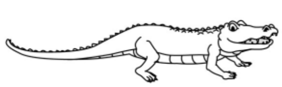
(b) P(sum of 11)

(c) P(sum > 8)

(d) P(sum < 5)

Solution:

You can use a two-way table to find these probabilities.



		Outcome of First Die					
		1	2	3	4	5	6
Outcome of Second Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(a) P(sum of 7) = $\frac{6}{36} = \frac{1}{6}$

(b) P(sum of 11) = $\frac{2}{36} = \frac{1}{18}$

(c) P(sum > 8) = P(9, 10, 11, 12) = $\frac{10}{36} = \frac{5}{18}$

(d) P(sum < 5) = P(2, 3, 4) = $\frac{6}{36} = \frac{1}{6}$

Now Try These:

1. A fair coin is tossed twice. How many possible outcomes are in the sample space?

2. Multiple Choice: A fair coin is tossed three times. What is the probability of at least one head?

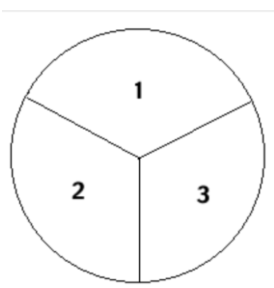
A. $\frac{7}{8}$

B. $\frac{3}{4}$

C. $\frac{3}{8}$

D. $\frac{1}{2}$

For questions 3-5, use the spinner below:



3. Multiple Choice: If the spinner is spun two times, what is the probability of getting a sum of 5?

A. $\frac{1}{3}$

B. $\frac{5}{9}$

C. $\frac{1}{2}$

D. $\frac{2}{9}$

4. Multiple Choice: If the spinner is spun two times, what is the probability of getting a sum greater than 3?

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 0

D. $\frac{2}{9}$

5. Multiple Choice: If the spinner is spun two times, what is the probability of getting an even sum?

A. $\frac{1}{3}$

B. $\frac{5}{9}$

C. $\frac{4}{9}$

D. $\frac{2}{9}$

6. Open Response

Jennifer and Ronald were playing a game using a six-sided fair die. Each of them rolled the die two times. Jennifer rolled a sum of seven and Ronald rolled a sum of 9. Jennifer stated that the probability of getting the sum of seven was higher than getting a sum of nine. Ronald stated that because nine is a larger number, the probability of the nine must be larger. Who is correct? Justify your answer.