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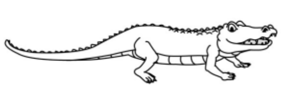


Table of Contents with Benchmarks

Translate Written and Mathematical Descriptions

Demonstrate an understanding of equality, the order of operations and equivalent numerical expressions

> Translate written real-world and mathematical descriptions into numerical expressions and numerical expressions into written mathematical descriptions.

Wylie reviews his understanding of a **numerical expression**. He remembers that a numerical expression is a combination of numbers with one or more operations (addition, subtraction, multiplication and division).

A numerical expression *does not* include an equal sign or an inequality symbol.

We can translate mathematical expressions into words and words into mathematical expressions just like we can write numbers in word form.

We know the number 0.35 can be also be written as "*thirty-five hundredths*" or even *"3 tenths and 5 hundredths."*

We can also write whole expressions this way.

The expression $3 + 4 - 2$ can be written in words as:

"3 increased by 4 decreased by 2" or

"3 plus 4 minus 2" or

"three increased by four minus 2"

We can continue writing phrases for this expression using different words that can represent addition and subtraction.

Each of the following phrases have been recorded as numerical expressions. Notice some of the words used in the phrases and the operations used in the expressions.

Phrases	Numerical Expression
5 more than 20	$20 + 5$
20 times 5	20×5
20 shared equally by 5	$20 \div 5$
20 decreased by 5	$20 - 5$

It is important to really understand what is happening in a word phrase before choosing operations and in a numerical expression before choosing words.

15 less 6 is expressed $15 - 6$

6 less than 15 is expression $15 - 6$, not $6 - 15$

This is because the phrase "less than" tells us that the first number is being taken from the second.

If Wylie has 5 less cards than his brother, we would see the number of cards his brother has and then subtract 5 to see the number Wylie has.

Example 1: Which of the following phrases represents the numerical expression $30 \div 5$?

a. 30 increased by 5
b. 5 divided by 30
c. 30 shared equally by 5
d. 5 groups of 30

Solution:

a. is incorrect because it reads $30 + 5$

b. is incorrect because it reads $5 \div 30$ rather than $30 \div 5$

c. is **correct** because it reads $30 \div 5$

d. is incorrect because it reads 5×30 or 30×5

Example 2: Write a numerical expression for each of the following phrases.

a. the product of 7 and 10
b. 25 increased by 30
c. the quotient of 81 and 9
d. the difference between 45 and 16
e. 64 less than 127
f. twice the amount of 235

Solution:

a. 7×10
b. $25 + 30$
c. $81 \div 9$
d. $45 - 16$
e. $127 - 64$
f. 2×235

Wylie knows that we can use symbols to group numbers together, such as parentheses ( ).

The numerical expression $4 \times (3 + 5)$ tells us to first add 3 and 5 and then multiply by 4.

We need to include the parentheses as we describe the expression with words.

We can write:

4 times the quantity of 3 plus 5 or

the sum of 3 and 5 is multiplied by 4 or

4 groups of the sum of 3 and 5

Just like before, we can continue writing expressions using different words to represent our operations and parentheses. The important thing is that it is clear that we have grouped the addition of 3 and 5 before multiplying by 4.

Example 3: Write a phrase for the numerical expression $(5 - 1) \div 2$.

Solution:

The difference of 5 and 1 is divided by 2.

The quantity of 5 minus 1 is divided by 2.

Example 4: Write a numerical expression to represent the phrase below:

36 less 6, then shared equally by 5

Solution:

Commas are another way to show that numbers are grouped:

36 less 6 means $36 - 6$ and the comma means this is a group so $(36 - 6)$.

Then shared equally by 5 tells us the group is divided by 5

$$(36 - 6) \div 5$$

Example 5: Which numerical expression represents the following phrase?

triple the sum of 1,456 and 2,589

a. $2 \times (1{,}456 + 2{,}589)$

b. $2 + (1{,}456 + 2{,}589)$

c. $3 \times (1{,}456 + 2{,}589)$

d. $3 \div (1{,}456 + 2{,}589)$

Solution:

Triple the amount is 3 times the amount.

Choice c is the correct answer.

Now Try These:

For 1-5, Open Response

1. Translate the numerical expression below into a written mathematical description:

$$15 - 0.2 - 0.5$$

2. Translate the numerical expression below into a written mathematical description:

$$3(14 - 3.8 + 0.9)$$

3. Translate the numerical expression below into a written mathematical description:

$$\frac{1}{4} \times \left(3 + \frac{1}{2}\right) - 5$$

4. Translate the numerical expression below into a written mathematical description:

$$(0.3 - 0.2) \div 5$$

5. Translate the numerical expression below into a written mathematical description:

$$(24 - 6) \times (36 \div 9)$$

6. Multiple Choice

Which of the following describes the numerical expression below?

$$5 \times (25 + 6)$$

A. Multiply 5 by 25, plus 6.

B. Multiply 5 by the product of 25 and 6.

C. Multiply 5 by the sum of 25 and 6.

D. Multiply 5 by the difference of 25 and 6.

7. Multiple Choice

Which expression represents the following mathematical description?

one-half of the product of 4 and 5 is added to the quotient of 63 and 7

A. $\frac{1}{2} \times 4 + 5 + 63 \div 7$

B. $\frac{1}{2} \times (4 + 5) \div (63 + 7)$

C. $\frac{1}{2} \times (4 + 5) + (63 \div 7)$

D. $\frac{1}{2} \times (4 \times 5) + (63 \div 7)$

For 8-13, Equation Editor

8. Translate the written mathematical description below into a numerical expression:

the sum of four and nine increased by fifteen

Evaluate Multi-Step Numerical Expressions

Demonstrate an understanding of equality, the order of operations and equivalent numerical expressions

Evaluate multi-step numerical expressions using the order of operations.

Wylie and his brother both evaluated the expression $6 + 5 \times 4$ below.

Wylie	Wylie's Brother
$6 + 5 \times 4$	$6 + 5 \times 4$
$6 + 20$	11×4
26	44

Notice that Wylie and his brother got different answers.

- Wylie said he did multiplication first because it is repeated addition, so it's like the 5 and 4 are grouped together.
- Wylie's brother said he did addition first because it came first reading from left to right.

So, which brother is right?

They both have a good idea, but in this problem, Wylie is correct!

He said that **multiplication is repeated addition** so it groups together 5 and 4. This is true. This means that if you see multiplication and addition in the same expression, you'll evaluate multiplication first, no matter where it is in the expression.

The same is true with multiplication and subtraction. The multiplication will come first.

Example 1: Evaluate the expressions below.

a. $4 + 6 \times 7$ **b.** $3 + 4 \times 5 + 8$

Solution:

a. $4 + 6 \times 7$

$4 + 42$

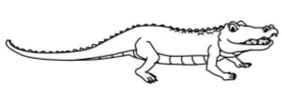
46

b. $3 + 4 \times 5 + 8$

$3 + 20 + 8$

$23 + 8$

31



Wylie's brother thinks this will apply to division, too, since it is repeated subtraction. He is correct!

If division occurs in the same expression as *addition* or *subtraction*, evaluate the division first.

Example 2: Evaluate the expressions below.

a. $3 + 20 \div 5$

b. $4 + 8 \div 2 - 3$

Solution:

a. $3 + 20 \div 5$

$3 + 4$

7

b. $4 + 8 \div 2 - 3$

$4 + 4 - 3$

$8 - 3$

5

Wylie noticed that in letter **b**, we completed the addition first and then the subtraction. We know that when we have an expression with addition and subtraction, we evaluate **from left to right**.

This is because addition and subtraction are related and just as important as each other.

Multiplication and division come *before* addition and subtraction only because they group the numbers associated with them (the factors or the dividend and divisor).

What happens when multiplication and division are in the same expression?

Wylie says we evaluate them **from left to right** because multiplication and division are related operations.

So, now we know that in an expression, if we see multiplication, division, addition, and subtraction, we start with multiplication or division depending on which comes first from left to right, then we work with addition and subtraction from left to right.

It is important to remember that we are working with **numerical expressions** and not **equations**. There are no equal or inequality symbols in an expression.

Example 3: Evaluate the expressions below.

a. $9 + 5 \times 2 - 3 \div 3$ **b.** $2 \times 4 + 8 \div 2 - 3$

Solution:

a. $9 + 5 \times 2 - 3 \div 3$

$9 + 10 - 3 \div 3$

$9 + 10 - 1$

$19 - 1$

18

b. $2 \times 4 + 8 \div 2 - 3$

$8 + 8 \div 2 - 3$

$8 + 16 - 3$

$24 - 3$

21

Notice that only one operation was computed each step. This is to keep track of our work. It is easier to make a mistake when we do too many steps at one time.

Wylie sees the expression $2 \times (5 - 3) + 1$ in a math book and thinks the parentheses also group numbers together.

Since parentheses group numbers together like multiplication and division, it is important to consider these groups before we do anything else.

This expression says *2 times the difference of 5 and 3 increased by 1*. This means we need to multiply 2 by the entire quantity in the parentheses.

Example 4: Evaluate the expression $2 \times (5 - 3) + 1$.

Solution:

$2 \times (5 - 3) + 1$

$2 \times (2) + 1$

$4 + 1$

5

Wylie and his brother think we've come up with some basic guidelines for evaluating expressions:

- Evaluate parentheses or other grouping symbols first, in order from left to right ( ), [ ], { }
- Evaluate multiplication and division next, in order from left to right
- Evaluate addition and subtraction last, in order from left to right

Evaluating expressions with decimals and fractions works the same way as evaluating expressions with whole numbers. We follow the same guidelines as we remember our rules for working with decimals and fractions.

Example 5: Evaluate the expressions below.

a. $3 \times (5.4 + 1.2) - 0.6$ **b.** $\frac{1}{2} \times (20 - 4) \div \frac{1}{4}$

Solution:

a. $3 \times (5.4 + 1.2) - 0.6$

$3 \times 6.6 - 0.6$

$19.8 - 0.6$

19.2

b. $\frac{1}{2} \times (20 - 4) \div \frac{1}{4}$

$\frac{1}{2} \times 16 \div \frac{1}{4}$

$8 \div \frac{1}{4}$

32

Example 6: The value of each expression has been given below each expression. Place parentheses in each expression so that the evaluation is correct.

a. $6 \times 12 - 8$

24

b. $12 \div 4 + 2$

2

c. $2 + 7 \times 7 - 2$

37

Solution:

a. $6 \times 12 - 8$

$6 \times (12 - 8)$

6×4

24

b. $12 \div 4 + 2$

$12 \div (4 + 2)$

$12 \div 6$

2

c. $2 + 7 \times 7 - 2$

$2 + 7 \times (7 - 2)$

$2 + 7 \times 5$

$2 + 35$

37

Now Try These:

For 1-10, Equation Editor

Determine the value of each numerical expression.

1. $(49 \div 7) + (63 \div 7)$

2. $25 + 15 \times 5$

3. $4 + 27 \div 3$

4. $8 - 8 \div 8$

5. $3.5 \times 2 + 2.5$

6. $6.3 \div 9 + 1.5 \times 7$

7. $\frac{1}{2} \times (6 \times 9) - 14$

8. $5 \div \frac{1}{4} + \left(\frac{2}{3} + 1\frac{2}{3}\right)$

9. $\frac{1}{2} \times (48 + 2) \times \left(40 \times \frac{1}{8}\right)$

10. $(6 + 38 \div 2) - 5 \times 5$

11. Multiselect

Select all of the expressions that have the same value as the expression below.

$$(5 \times 3) + (28 \div 7)$$

A. $8 + 6 + 5 \times 1$

B. $5 + 2 + 56 \div 8$

C. $5 + 64 \div 8 + 6$

D. $1 \times 0 + 12 + 6$

E. $15 - 4 + 4 \times 2$

For 12-14, GRID

12. Place parentheses in the expression $6 + 7 \times 8$ so that the expression simplifies to 104.

$$6 + 7 \times 8$$

( )

13. Place parentheses in the expression $9 - 4 \times 15 \div 3$ so that it simplifies to 25.

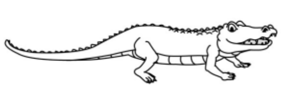
$$9 - 4 \times 15 \div 3$$

( )

14. Place parentheses in the expression $6 + 4 \times 3 - 1$ so that the expression simplifies to 20.

$$6 + 4 \times 3 - 1$$

( ) ( )



Working with Numerical Patterns

Analyze patterns and relationships between inputs and outputs

MA.5.AR.3.1 Given a numerical pattern, identify and write a rule that can describe the pattern as an expression.

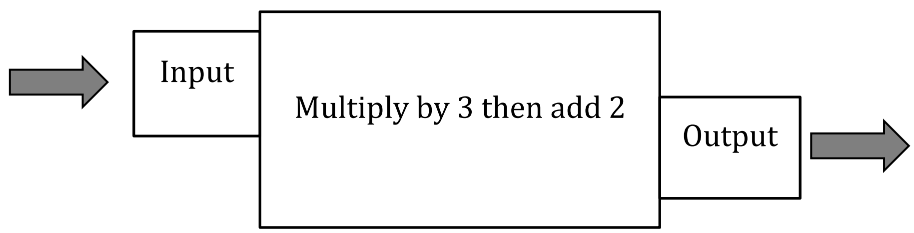
MA.5.AR.3.2 Given a rule for a numerical pattern, use a two-column table to record the inputs and outputs.

A **numerical pattern** is a sequence of numbers that follows a predictable pattern, or a **rule**. Often, a rule is applied to a number called an **input** that generates a new number called an **output**.

Think about the rule "multiply by 3 then add 2."

This rule tells us to multiply an input by 3 and then add 2 to the product. The outcome is our output.

The function machine below shows the input and output relationship for the rule above:



The input 0 is what we multiply by 3	The input 1 is what we multiply by 3	The input 2 is what we multiply by 3	The input 3 is what we multiply by 3
$3 \times 0 + 2$	$3 \times 1 + 2$	$3 \times 2 + 2$	$3 \times 3 + 2$
$0 + 2$	$3 + 2$	$6 + 2$	$9 + 2$
2	5	8	11

Wylie knows he can use tables to represent quantities that are related to each other. We can use a table this way to represent rules for numerical patterns.

Input (x)	0	1	2	3
Output	2	5	8	11

Notice that this table looks like a table used to plot points on a graph. That means the values in this table can be written as ordered pairs that could be plotted on a coordinate plane!

The input is the x-coordinate and the output is the y-coordinate.

$$(0, 2), (1, 5), (2, 8), (3, 11)$$

Something else we can see from the table is that rules can be written as expressions where the variable x represents the input. The output is the value after we evaluate the expression for a specific input.

The expression that represents this rule is $3x + 2$.

In this expression, 3 is our **coefficient**, the number before the variable. It tells us to multiply the unknown amount by 3.

2 is our **constant**, it isn't changed by the variable.

Example 1: Create a table to show the rule $5x - 1$ for the inputs $x = 1$, $x = 2$, $x = 3$, and $x = 4$. Then, give the ordered pairs that could be graphed on the coordinate plane.

Solution:

The input 1 is what we substitute for x. $5x - 1$ $5 \times 1 - 1$ $5 - 1$ 4	The input 2 is what we substitute for x. $5x - 1$ $5 \times 2 - 1$ $10 - 1$ 9	The input 3 is what we substitute for x. $5x - 1$ $5 \times 3 - 1$ $15 - 1$ 14	The input 4 is what we substitute for x. $5x - 1$ $5 \times 4 - 1$ $20 - 1$ 19

Input (x)	1	2	3	4
Output	4	9	14	19

(1, 4), (2, 9), (3, 14), (4, 19)

Example 2: What is the missing value in the table below? The table represents the rule $30 - 4x$.

Input (x)	0	1	2	3
Output	?	26	22	28

Solution:

Substitute 0 for x in the expression: $30 - 4x$

$$30 - 4(0)$$

$$30 - 0$$

$$30$$

The missing value is 30.

Example 3: Adam's computer makes tables whenever the user tells it a rule. Adam tells his computer to create a table using the rule $5 + 7x$. Unfortunately, the computer malfunctioned when it created the table below and only some of the outputs are correct. Identify which outputs are incorrect and complete the table correctly.

Input (x)	0	1	2	3
Output	12	12	17	22

Solution:

The outputs for 0, 2, and 3 are incorrect. The correct table is:

Input (x)	0	1	2	3
Output	5	12	19	26

Just as we can create a table from a rule, we can write a rule from a table or a list of numbers in a pattern.

Example 4: Write a rule for the values in the input/output table below.

Input (x)	0	1	2	3
Output	4	7	10	13

Solution:

One rule that can be written for this table is *multiply the input by 3 then add 4.*

Written as an expression this is $3x + 4$.

We are not always given the inputs when we are determining a rule. Sometimes, we are just given the outputs in the pattern.

Consider the pattern 6, 8, 10, 12.

We can create a table with possible input values to determine a rule. Since we do not know the input values, we will assign them ourselves and check for a rule.

Let's choose the inputs $x = 0$, $x = 1$, $x = 2$, $x = 3$:

Input (x)	0	1	2	3
Output	6	8	10	12

With the inputs we selected here we can see the rule *multiply by 2 then add 6.*

$2x + 6$	$2x + 6$	$2x + 6$	$2x + 6$
$2 \times 0 + 6$	$2 \times 1 + 6$	$2 \times 2 + 6$	$2 \times 3 + 6$
$0 + 6$	$2 + 6$	$4 + 6$	$6 + 6$
6	8	10	12

We can see this rule works for all the output values we have.

What about if we use the inputs in the table below?

Input (x)	1	2	3	4
Output	6	8	10	12

Does the rule change with these chosen inputs?

The rule *multiply by 2 then add 4* looks like it would work. Let's try it!

$2x + 4$	$2x + 4$	$2x + 4$	$2x + 4$
$2 \times 1 + 4$	$2 \times 2 + 4$	$2 \times 3 + 4$	$2 \times 4 + 4$
$2 + 4$	$4 + 4$	$6 + 4$	$8 + 4$
6	8	10	12

So, we can see this rule also works when we adjust the inputs.

Wylie sees that we can create many rules for a numerical pattern depending on the inputs we use.

Example 5: Give a rule for the pattern 3, 6, 9, 12 for the inputs $x = 1, 2, 3,$ and 4.

Solution:

Input: 1	Input: 2	Input: 3	Input: 4
Output: 3	Output: 6	Output: 9	Output: 12
Rule: $3x$	Rule: $3x$	Rule: $3x$	Rule: $3x$
$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$

So the rule is $3x$ or *multiply by 3*.

Example 6: Give a rule for the pattern 3, 6, 9, 12 for the inputs $x = 0, 1, 2, 3$.

Solution:

Input: 0	Input: 1	Input: 2	Input: 3
Output: 3	Output: 6	Output: 9	Output: 12
Rule: $3x + 3$	Rule: $3x + 3$	Rule: $3x + 3$	Rule: $3x + 3$
$3 \times 0 + 3$	$3 \times 1 + 3$	$3 \times 2 + 3$	$3 \times 3 + 3$
$0 + 3$	$3 + 3$	$6 + 3$	$9 + 3$
3	6	9	12

The rule is $3x + 3$ or *multiply by 3, then add 3.*

Now Try These:

For 1-6, Equation Editor

For 1-3, write an expression that can be a rule for the terms 5, 8, 11, 14 with the inputs given in each problem.

1. $x = 1, 2, 3, 4$

2. $x = 2, 3, 4, 5$

3. $x = 0, 1, 2, 3$

For 4-6, write an expression that can be a rule for the terms 0, 4, 8, 12 with the inputs given in each problem.

4. $x = 1, 2, 3, 4$

5. $x = 0, 1, 2, 3$

6. $x = 2, 3, 4, 5$

7. Open Response

Write a mathematical description for a rule the matches the terms 1, 5, 9, 13. Include the inputs for your rule.

For 8-10, Table Item

For 8-10, Create a table for the rule given with the inputs listed.

8. $5x - 3$

Input (x)	1	2	3	4
Output				

9. $3 + x$

Input (x)	0	1	2	3
Output				

10. $10x$

Input (x)	3	4	5	6
Output				

For 11-14, Table Item

For 11-14, Determine the missing values in the tables using the given rule.

11. Rule: $25 - 2x$

Input (x)	0	1	2	3
Output	?	23	?	19

12. Rule: $14 + 3x$

Input (x)	1	2	3	4
Output	17	?	?	26

13. Rule: $6x - 4$

Input (x)	2	3	4	5
Output	?	14	?	26

14. Rule: $7x$

Input (x)	5	6	7	8
Output	35	42	?	?