Table of Contents with Standards

Domain 1

Circles

Prove all circles are similar

MAFS.912.G-C.1.1
Prove that all circles are similar.

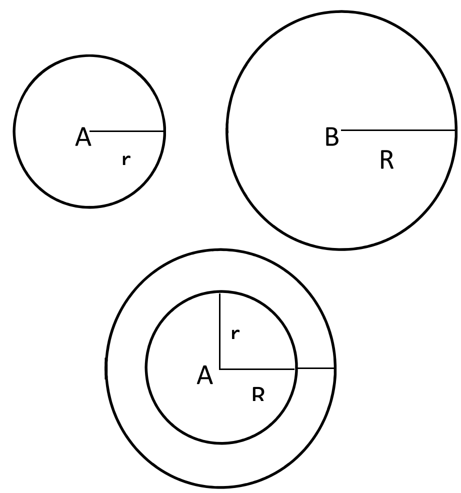
REVIEW:

A circle is a set of points in a plane equidistant from a given point called the center. This constant distance is called the radius of the circle.

Two geometric figures are similar if one or more similarity transformations (translation, reflection, rotation or dilation) are applied to map one figure to the other.

To prove any two circles are similar, only translation, dilation, or both is required.

In the figure, we have circle A with radius r and circle B with radius R, where $R > r$.



To prove that circle A is similar to circle B:

1. Translate the center of circle A over the center of circle B, so both circles have the same center. Let (x, y) be the center of circle A, then the algebraic description of the translation can be written as

   $(\boldsymbol{x}, \boldsymbol{y}) \rightarrow (\boldsymbol{x} + \boldsymbol{a}, \boldsymbol{y} + \boldsymbol{b})$**.**

2. Dilate circle A to increase its size equal to circle B.

   The scale factor to increase circle A will be $\frac{R}{r}$.

   Therefore, the similarity transformation for circle A and B is a translation followed by a dilation with scale factor $\frac{R}{r}$.

Example 1:

Describe how circle A with center (0, 0) and a radius of 2 units can be transformed into circle B with center (2, 3) and a radius of 6 units.

Solution:

To transform circle A to the larger circle B, find the translation for the center and the scale factor for the radius.

1. Translate the center of circle A 2 units right and 3 units up.

2. To enlarge circle A to the same radius as circle B, the scale factor is the ratio of the radii, $\frac{6}{2} = 3$.

Example 2:

Circle A with center (-4, 1) and a radius 4 units is similar to circle B with center (2, 3) and a radius of 6 units. Write an algebraic description for the transformations.

Solution:

To transform circle A to the larger circle B, find the translation for the center and the scale factor for the radius.

1. Translate the center of circle A 6 units right and 2 units up.

2. To enlarge circle A to the same radius as circle B, the scale factor is the ratio of the radii $\frac{6}{4} = \frac{3}{2}$.

The algebraic description of transformations can be written as,

Translation: $(x, y) \to (x + 6, y + 2)$

Dilation: $D_{\frac{3}{2}}$

Example 3:

Circle X with center (3, 5) and a radius of 8 units is similar to circle Y with center (-1, -3) and a radius of 2 units. Write the similarity transformations algebraically.

Solution:

To transform circle X to the smaller circle Y, find the translation for the center and the scale factor for the radius.

1. Translate the center of circle X 4 units left and 8 units down.

2. To reduce circle X to the same radius as circle Y, the scale factor is the ratio of the radii $\frac{2}{8} = \frac{1}{4}$.

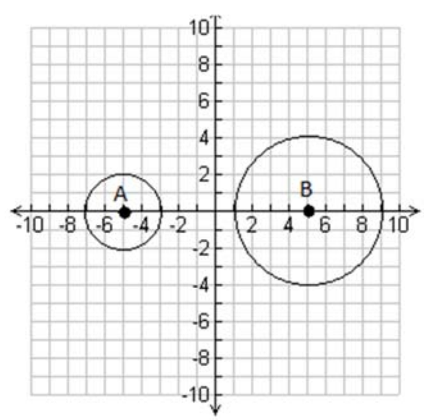
$Translation: (x, y) \rightarrow (x - 4, y - 8)$

$Dilation: D_{\frac{1}{4}}$

Example 4:

Write the algebraic description for the similarity transformations of circle A and B. One of the transformations is centered at B.

Solution:



In the first step, we will translate the center of circle A to the center of circle B.

Move the center of circle A 10 units to the right to coincide with the center of circle B. This can be written algebraically as: $(x, y) \rightarrow (x + 10, y)$.

Now enlarge circle A, so its radius is the same length as the radius of circle B. The scale factor can be calculated as the ratio of the radii.

Radius of circle B = 8

Radius of circle A = 4

Scale Factor = $\frac{8}{4} = 2$. This can be algebraically written as D_2.

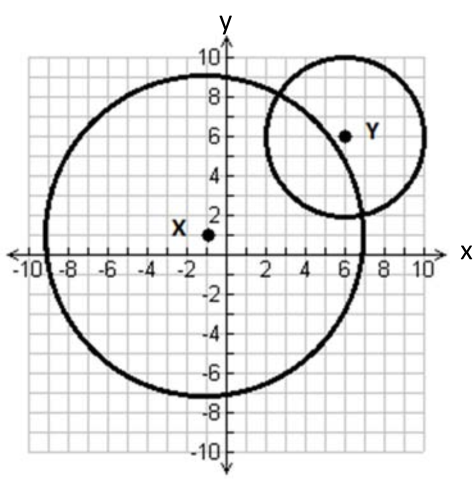
Now Try These:

For 1 - 3, Equation Editor

1. Write an algebraic description for the similarity transformation for a circle centered at (-2, -1) with a radius of 3 units to a circle centered at (5, 3) with a radius of 7 units.

2. Joe draws a circle with center (-5, 2) and a radius of 4 units. If the scale factor is $\frac{5}{2}$, what is the radius of a similar circle with the same center?

3. Write an algebraic description for the similarity transformation which maps circle X onto circle Y.

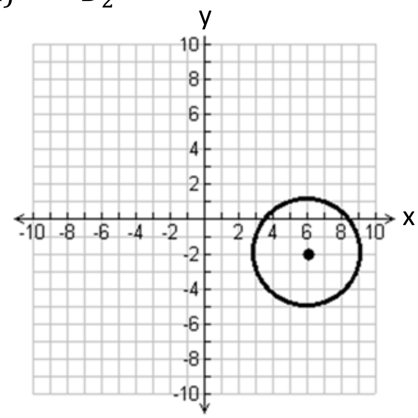


For 4 – 5, GRID

4. Apply the transformations to the given circle and draw the similar circle on the grid below.

i) $(x, y) \rightarrow (x - 5, y + 2)$

ii) D_2

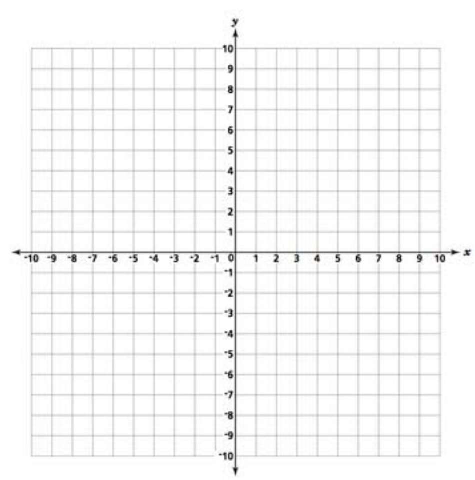


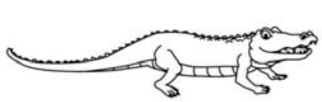
5. Alicia and Anna were asked to draw similar circles. Alicia drew a circle centered at the origin with a radius of 8 units. Anna applied the transformations below to create a circle similar to Alicia's circle.

i) $(x, y) \rightarrow (x - 3, y2)$

ii) $D_{\frac{1}{4}}$

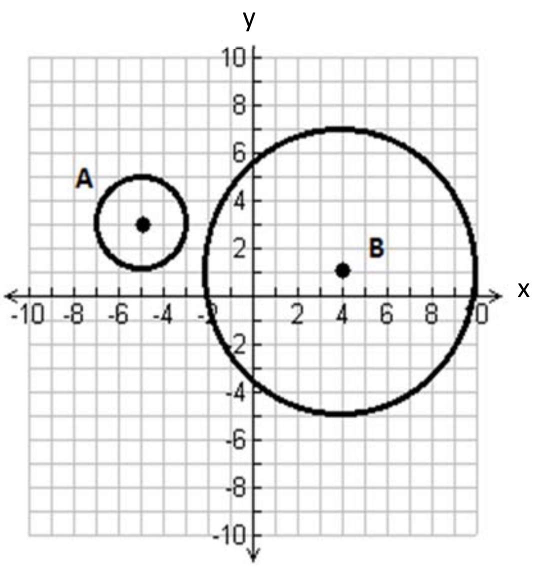
Draw Alicia's and Anna's circles.





6. Hot Text

Drag and drop all the correct transformation statements which support the similarity of circles A and B into the box below.



A. $(x, y) \to (x + 9, y - 2);\ D_3$

B. $(x, y) \to (x - 9, y + 2);\ D_3$

C. $(x, y) \to (x - 9, y + 2);\ D_{\frac{1}{3}}$

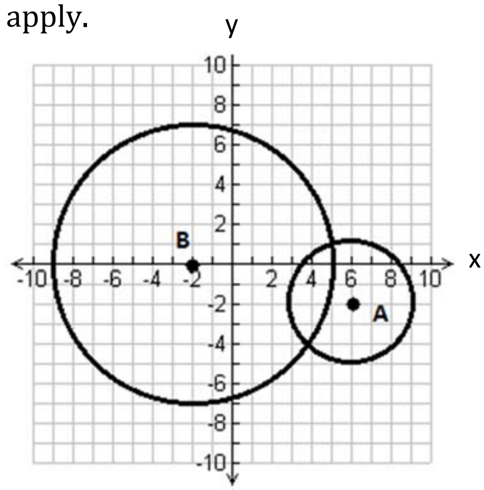
D. $(x, y) \to (x + 9, y - 2);\ D_{\frac{1}{3}}$

E. $(x, y) \to (x + 2, y - 9);\ D_{\frac{1}{3}}$

F. $(x, y) \to (x + 2, y - 9);\ D_3$

For 7 - 8, Multiselect

7. If circle A is transformed onto circle B, which statements can be used to prove the similarity of circles A and B? Select all that apply.



A. $(x, y) \to (x + 8, y - 2)$
B. $(x, y) \to (x - 8, y + 2)$
C. $(x, y) \to (x - 8, y - 2)$
D. $(x, y) \to (x + 8, y + 2)$
E. D_7
F. $D_{\frac{7}{3}}$

8. Select all the transformation statements which prove that circle P and circle Q are similar.
Circle P: Center (0, -3), Radius = 8
Circle Q: Center (-5, 3), Radius = 5

A. $(x, y) \to (x - 5, y + 6);\ D_{\frac{5}{8}}$

B. $(x, y) \to (x + 5, y - 5);\ D_{\frac{5}{8}}$

C. $(x, y) \to (x - 5, y + 5);\ D_{\frac{8}{5}}$

D. $(x, y) \to (x + 5, y - 6);\ D_{\frac{8}{5}}$

E. $(x, y) \to (x + 6, y - 5);\ D_{\frac{8}{5}}$

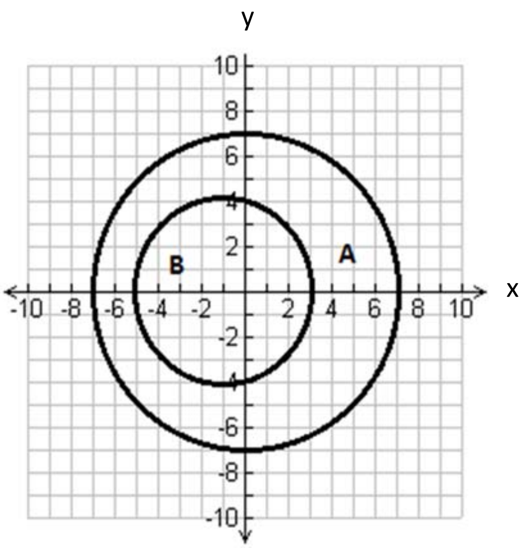
F. $(x, y) \to (x - 6, y + 5);\ D_{\frac{5}{8}}$

For 9 - 10, Open Response

9. Prove that the circles below are similar by using transformation statements.

Circle S: Center (-2, 8), Radius = 4
Circle T: Center (0, 4), Radius = 9

10. Prove that circles A and B are similar.



For 11 – 12, Table Item

11. Complete the table by using the similarity transformations that map circles A onto circle B.

i) $(x, y) \rightarrow (x - 3, y + 2)$
ii) $D_{\frac{1}{2}}$

	Center	Radius
Circle A	(-2, 3)	5
Circle B		

12. Complete the table by using the similarity transformations that map circles X onto circle Y.

i) $(x, y) \rightarrow (x + 5, y - 10)$
ii) D_3

	Center	Radius
Circle X	(-8, 7)	4
Circle Y		

For 13 – 14, Multiple Choice

13. Select the correct similarity transformations for the given circles.

Circle X: Center (2, -5), Radius = 3
Circle Y: Center (-3, -3), Radius = 1

A. $(x, y) \rightarrow (x + 5, y - 2);\ D_{\frac{1}{3}}$

B. $(x, y) \rightarrow (x - 5, y + 2);\ D_3$

C. $(x, y) \rightarrow (x - 5, y + 2);\ D_{\frac{1}{3}}$

D. $(x, y) \rightarrow (x + 5, y + 2);\ D_3$

14. Select the correct similarity transformations for the given circles.

Circle A: Center (7, 1), Radius = 2
Circle B: Center (-1, -2), Radius = 5

A. $(x, y) \rightarrow (x - 8, y - 3);\ D_{\frac{2}{5}}$

B. $(x, y) \rightarrow (x - 8, y - 3);\ D_{\frac{5}{2}}$

C. $(x, y) \rightarrow (x + 8, y + 3);\ D_{\frac{5}{2}}$

D. $(x, y) \rightarrow (x - 8, y + 3);\ D_{\frac{5}{2}}$

Everglades K-12 Publishing's Mathematics Florida Standards Geometry
Domain 1 - Formative Assessment 1

Circles

Formative Assessment 1

Solve and answer all of the problems on this assessment. Select the best answer for each of the Multiple Choice, Multiselect, and Matching Item problems. Complete the Equation Editor, Table Item, Open Response, Hot Text, Editing Task Choice and Graphic Response Item Display (GRID) problems.

Multiple Choice

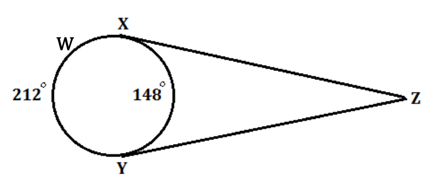
1. Which similarity transformation correctly maps circle A onto circle B? MAFS.912.G-C.1.1

Circle A: center (2, 5), Radius = 3
Circle B: Center (-3, 2), Radius = 7

A. $(x, y) \to (x - 5, y + 3);\ D_{\frac{7}{3}}$
B. $(x, y) \to (x - 5, y - 3);\ D_{\frac{7}{3}}$
C. $(x, y) \to (x + 5, y + 3);\ D_{\frac{3}{7}}$
D. $(x, y) \to (x - 5, y - 3);\ D_{\frac{3}{7}}$

For 2 – 4, Multiselect

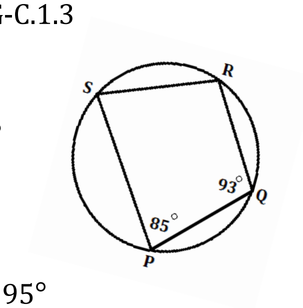
2. $\widehat{XWY} = 212°$ and $XY = 148°$. Which of the following statements are true for $m\angle XZY$? Select all that apply.
MAFS.912.G-C.1.2



A. $64°$
B. $\frac{1}{2}(212° - 148°)$
C. $(212° - 148°)$
D. $\frac{1}{2}(64°)$
E. $\frac{1}{2}(212° + 148°)$
F. $32°$

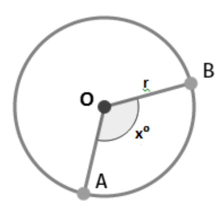
3. PQRS is a quadrilateral inscribed in a circle. Find $m\angle R$ and $m\angle S$.
MAFS.912.G-C.1.3

m∠P = 85°
$m\angle Q = 95°$



A. $m\angle R = 95°$
B. $m\angle R = 85°$
C. $m\angle R = 87°$
D. $m\angle S = 93°$
E. $m\angle S = 95°$
F. $m\angle S = 87°$

4. Given: a circle with center O, a radius of 8 cm and the measure of central angle AOB = 55°. Which expressions represent the area of sector AOB? MAFS.912.G-C.2.5



A. $\frac{1}{2}(8)^2(55)$
B. $\frac{1}{2}(55)^2(8)$
C. $\frac{1}{2}(8)^2\left(\frac{\pi}{180}\right)(55)$
D. $\frac{1}{2}(8)^2\left(\frac{180}{\pi}\right)(55)$
E. $(0.0087)(8)^2(55)$
F. $(28.6479)(8)^2(55)$

For 5 – 7, Matching Item

5. Match the columns to the correct properties.
MAFS.912.G-C.1.3

Center of circumscribed circle	Circumscribed Circle
Center of inscribed circle	Inscribed Circle
Circle is touching the vertices of a triangle.	Circumcenter
Circle is touching sides of a triangle.	Incenter

6. Match the columns.
MAFS.912.G-C.1.2

Central Angle	half the measure of intercepted arc
Inscribed Angle	right angle
Inscribed angle on the diameter	measure of intercepted arc

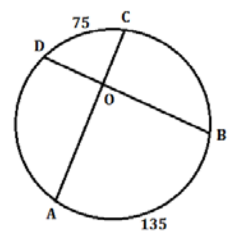
7. Match the columns.
MAFS.912.G-C.2.5

Exterior angle of a circle	$\frac{1}{2}r^2\theta$
Interior angle of a circle made by two intersecting chords	Half the difference of the intercepted arcs.
Length of an arc	$r\theta$
Area of a sector of a circle	Half the sum of the intercepted arcs

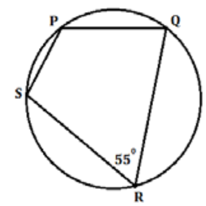
For 8 – 10, Equation Editor

8. A circle has a center at (0, 3) and a radius of 6 units. If a similar circle has the same center and is dilated by a scale factor $\frac{3}{2}$, what is the length of the similar circle's radius?
MAFS.912.G-C.1.1

9. Using the figure to write and solve an equation to find $m\angle AOB$.
MAFS.912.G-C.1.2

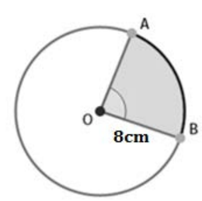


10. Inscribed quadrilateral PQRS is shown in the figure. Find $m\angle P$, if $m\angle R = 55°$. MAFS.912.G-C.1.3



For 11 – 13, Table Item

11. Complete the table using the following information. Circle O has a radius of 8 cm and $m\angle AOB = 75°$. MAFS.912.G-C.2.5



Length of Arc AB	?
Area of Sector AOB	?

12. In circle O, $\angle AOC$ measures 60°. Complete the table. Point P is on the circle, but not on arc AC.
MAFS.912.G-C.2.5

$m\angle ABC$	?
m(arc AC)	?

13. Fill in the table based on the similarity transformations that maps circle X onto circle Y.
MAFS.912.G-C.1.1

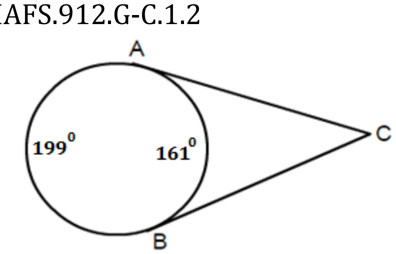
i) $(x, y) \to (x - 3, y + 7)$
ii) $D_{\frac{1}{3}}$

	Center	Radius
Circle X	(-5, -7)	27
Circle Y		

For 14 – 16, Open Response

14. Prove the given circles are similar.
Circle S: Center (3, -2); Radius = 5
Circle T: Center (0, -8); Radius = 8
MAFS.912.G-C.1.1

15. What is the $m\angle ACB$?
MAFS.912.G-C.1.2

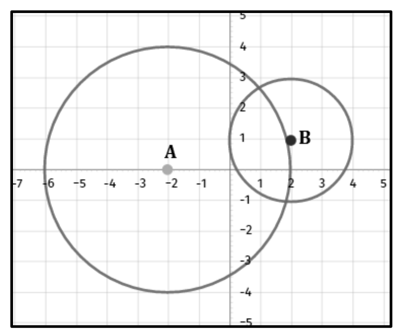


16. A sector of a circle has an area of 84 square inches. The central angle intercepted by the sector is 60°.
MAFS.912.G-C.2.5

a. Find the length of the circle's radius.
b. What is the length of the arc intercepted by the given central angle?

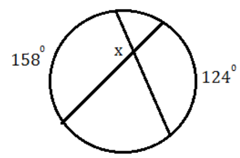
For 17 – 19, Hot Text

17. Drag and drop all the transformation statements which prove circles A and B are similar into the box below.
MAFS.912.G-C.1.1



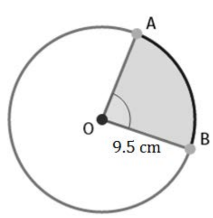
- $(x, y) \to (x + 4, y - 1);\ D_2$
- $(x, y) \to (x - 4, y + 1);\ D_2$
- $(x, y) \to (x + 4, y + 1);\ D_{\frac{1}{2}}$
- $(x, y) \to (x + 4, y - 1);\ D_{\frac{1}{2}}$
- $(x, y) \to (x + 4, y - 1);\ D_{\frac{1}{2}}$
- $(x, y) \to (x - 4, y - 1);\ D_2$

18. Drag and drop the expressions equal to the value of x into the box below. MAFS.912.G-C.1.2



- $17°$
- $\frac{1}{2}(158° - 124°)$
- $(158° - 124°)$
- $\frac{1}{2}(282°)$
- $\frac{1}{2}(158° + 124°)$
- $141°$

19. In the given figure, $OB = 9.5\ cm$ and $m\angle AOB = \frac{3\pi}{2}\ radians$.
Drag and drop the correct expressions to find the area of the sector AOB into the box below.
MAFS.912.G-C.2.5



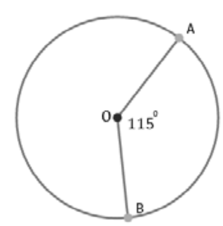
- $A = \frac{1}{2}(9.5)^2\left(\frac{3\pi}{2}\right)\left(\frac{\pi}{180}\right)$
- $A = \frac{1}{2}(9.5)^2\left(\frac{3\pi}{2}\right)\left(\frac{180}{\pi}\right)$
- $A = \frac{1}{2}(9.5)^2\left(\frac{3\pi}{2}\right)$
- $A = 12{,}183.75\ cm^2$
- $A = 3.7\ cm^2$
- $A = 212.65\ cm^2$

For 20 – 22, GRID

20. A circle is given with center (-3, -3) and a radius of 8 units. Apply the following transformations to the given circle and draw a similar circle on the grid.
MAFS.912.G-C.1.1

i) $(x, y) \to (x + 3, y + 2)$
ii) $D_{\frac{1}{2}}$

21. On the circle O below, draw an inscribed angle that measures half of the measure central angle AOB.
MAFS.912.G-C.1.2

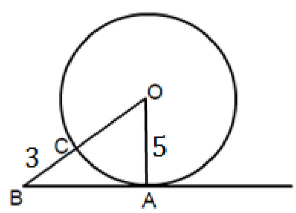


22. Refer to the image in Question 21. Draw a similar circle with the given angle of 115° as the constant of proportionality.
MAFS.912.G-C.2.5

For 23 – 24, Editing Task Choice

Choose the value or expression to correctly fill in the highlighted area.

23. In the given figure, AB = $\sqrt{5^2 - 3^2}$, $\sqrt{8^2 - 5^2}$, $\sqrt{5^2 - 2^2}$ or $\sqrt{5^2 + 8^2}$.
MAFS.912.G-C.1.2



24. The radius of a circle is 10 units. The area of the sector bounded by a 120° arc is $10.47, 26.18, 104.72$ *or* 20.94 $units^2$, if the circle is dilated by a scale factor of 0.5?
MAFS.912.G-C.2.5