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Table of Contents with Benchmarks

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Make formal geometric constructions with a variety of tools and methods.

Use properties and theorems related to circles.

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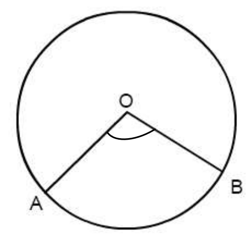
Arc and Angle Measures

Use properties and theorems related to circles.

Solve mathematical and real-world problems involving the measures of arcs and related angles.

Central Angle

An angle whose vertex is the center of the circle and whose sides are radii of the circle is called a central angle.

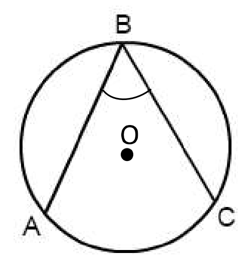


In the figure, $\angle AOB$ is a central angle. The measure of a central angle is equal to the measure of the arc it intercepts.

$$m\angle AOB = m\widehat{AB}$$

Inscribed Angle

An inscribed angle of a circle is an angle made by any two chords of the circle and whose vertex a point on the circle.

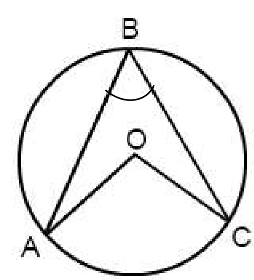


Inscribed angle $\angle ABC$ is shown in circle O.

Inscribed Angle Theorem

The inscribed angle theorem states that an inscribed angle is equal to one half the measure of its intercepted arc.

$$m\angle ABC = \frac{1}{2} m\widehat{AC}$$



Additionally, if a central angle is equal to its intercepted arc, then an inscribed angle, intersecting the same arc as the central angle, will be equal to half the measure of the central angle.

$$m\angle ABC = \frac{1}{2}(m\angle AOC)$$

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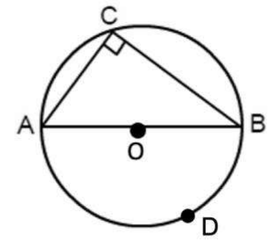
Inscribed Angle on Diameter

When an inscribed angle intercepts a semicircle, the inscribed angle is a right angle.

$$m\angle ACB = \frac{1}{2}\left(m\widehat{ADB}\right) = \frac{1}{2}(180°) = 90°$$



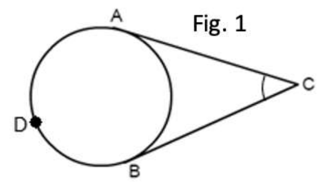
Exterior Angles of a Circle

An exterior angle of a circle is an angle formed by two tangents, a tangent and a secant, or two secants that intersect outside a circle.

The exterior angle of a circle is equal to half the difference of the measures of the intercepted arcs.

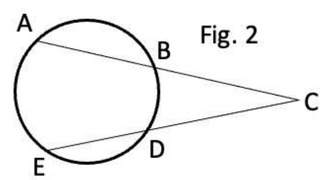
In figure 1, $\angle ACB$ is the exterior angle between the two tangents $\overline{AC}$ and $\overline{BC}$. The intercepted arcs are $\widehat{ADB}$ and $\widehat{AB}$.

$$m\angle ACB = \tfrac{1}{2}(m\widehat{ADB} - m\widehat{AB})$$



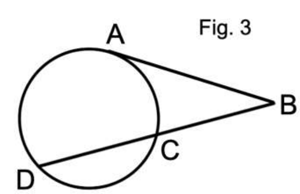
In figure 2, $\angle ACE$ is the exterior angle between the two secants $\overline{AC}$ and $\overline{EC}$. The intercepted arcs are $\widehat{AE}$ and $\widehat{BD}$.

$$m\angle ACE = \tfrac{1}{2}(m\widehat{AE} - m\widehat{BD})$$



In figure 3, $\angle ABD$ is the exterior angle between tangent $\overline{AB}$ and secant $\overline{BD}$. The intercepted arcs are $\widehat{AD}$ and $\widehat{AC}$.

$$m\angle ABD = \tfrac{1}{2}(m\widehat{AD} - m\widehat{AC})$$



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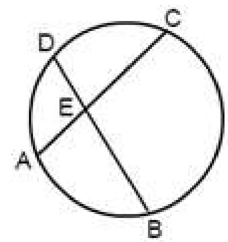
Angles formed inside the circle by two intersecting chords



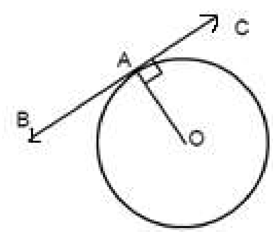
When two chords intersect inside a circle, four angles are formed. In the figure, we can see chords $\overline{AC}$ and $\overline{BD}$ are intersecting inside the circle at point E. Two pairs of vertical angles are formed at point E.

Each angle measure is equal to half the sum of the measures of the intercepted arcs.

$$m\angle DEA = m\angle BEC = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$$

$$m\angle DEC = m\angle AEB = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$$

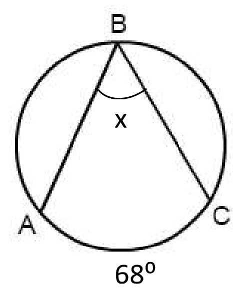
Radius and Tangent of a Circle



The radius of a circle is always perpendicular to a tangent line at the point of tangency.

In the figure, the radius $\overline{OA}$ is perpendicular to the tangent $\overleftrightarrow{BC}$ at point A.

Example 1:



Find $m\angle ABC$, represented by x, if $m\widehat{AC} = 68°$.

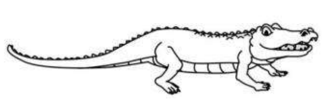
Solution:

$\angle ABC$ is an inscribed angle. An inscribed angle is equal to half the measure of the intercepted arc.

$m\angle ABC = \frac{1}{2}(m\widehat{AC})$

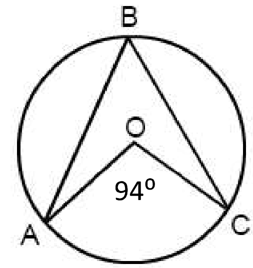
$= \frac{1}{2}(68°)$

$= 34°$



Example 2:

If $m\angle AOC = 94°$, find $m\angle ABC$ and $m\widehat{AC}$.



Solution:

$\angle ABC$ is an inscribed angle and $\angle AOC$ is a central angle. If an inscribed angle and a central angle intersect the same arc, then the measure of the inscribed angle is equal to half the measure of the central angle.

$$m\angle ABC = \frac{1}{2}(m\angle AOC)$$

$$= \frac{1}{2}(94°)$$

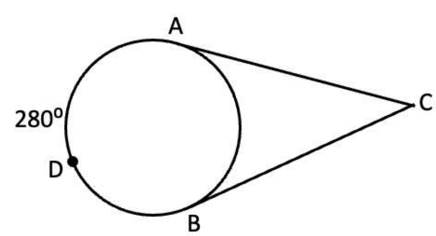
$$= 47°$$

The measure of the arc intercepted by the central angle is always equal to the measure of the central angle.

$$m\widehat{AC} = m\angle AOC = 94°$$

Example 3:

In the given figure, find $m\angle ACB$.



Solution:

$\angle ACB$ is an exterior angle. The measure of an exterior angle of a circle is equal to half the difference of the measures of its intercepted arcs.

$m\widehat{AB}$ = 360° – 280° = 80°

$$m\angle ACB = \frac{1}{2}\ (m\widehat{ADB} - m\widehat{AB})$$

$$= \frac{1}{2}(280° - 80°)$$

$$= \frac{1}{2}(200°) = 100°$$

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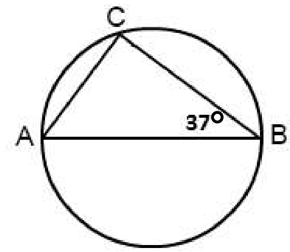
Example 4:

Find $m\angle BAC$, if $m\angle ABC = 37°$ and $\overline{AB}$ is a diameter of the circle.



Solution:

An inscribed angle intercepting the diameter of the circle is a right angle, so $m\angle ACB = 90°$

Consider the ΔACB.

$m\angle ACB + m\angle ABC + m\angle BAC = 180°$

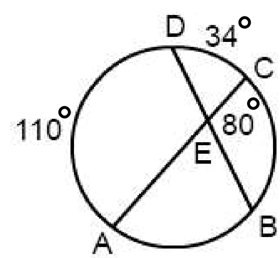
$90° + 37° + m\angle BAC = 180°$

$127° + m\angle BAC = 180°$

$m\angle BAC = 53°$

Example 5:

In the figure, $\widehat{AD} = 110°$, $m\angle BEC = 80°$, $m\widehat{DC} = 34°$. Find $m\widehat{BC}$ and $m\widehat{AB}$.



Solution:

The measure of an interior angle made by two intersecting chords is equal to half the sum of the measures of its intercepted arcs.

$m\angle BEC = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

$80° = \frac{1}{2}(110° + m\widehat{BC})$

$160° = 110° + m\widehat{BC}$

$m\widehat{BC} = 50°$

$m\widehat{AB} + m\widehat{BC} + m\widehat{CD} + m\widehat{DA} = 360°$

$m\widehat{AB} + 50° + 34° + 110° = 360°$

$m\widehat{AB} + 194 = 360°$

$m\widehat{AB} = 166°$

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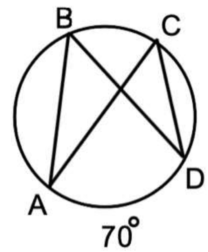
Example 6:

In the figure, $\widehat{AD} = 70°$. Find $m\angle ABD$ and $m\angle ACD$.



Solution:

$\angle ABD$ and $\angle ACD$ are both inscribed angles with the same intercepted arc, $\widehat{AD}$. The measure of an inscribed angle is equal to half the measure of its intercepted arc.

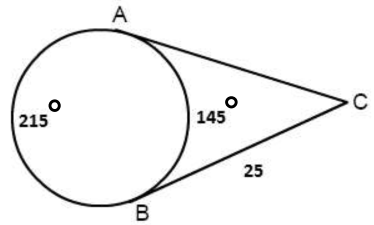
$$m\angle ABD = m\angle ACD = \frac{1}{2}(70°) = 35°$$

Note: Inscribed angles that intercept the same arc will always be congruent.

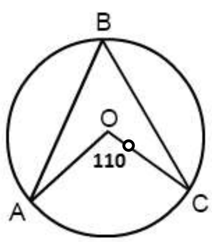
Now Try These:

For 1 – 2, Equation Editor

1. Use the figure below. Write and solve an equation to find $m\angle ACB$.

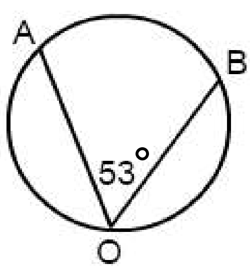


2. If $m\angle AOC = 110°$, what is the $m\angle ABC$ in the diagram below?



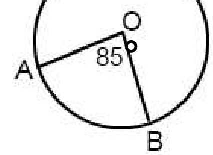
For 3 – 4, GRID

3. In the given figure, the measure of inscribed angle AOB is 53°. Draw another inscribed angle in the same circle congruent to $\angle AOB$.



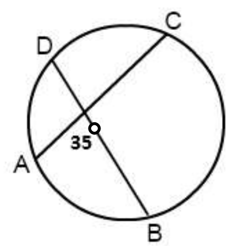
4. ∠AOB is a central angle and $m\angle AOB = 85°$.

Use circle O to draw an inscribed angle whose measure is half the measure of ∠AOB.



For 5 – 6, Multiple Choice

5. Based on the figure below, which statement is correct?



A. $35° = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

B. $35° = \frac{1}{2}(m\widehat{AB} - m\widehat{DC})$

C. $35° = \frac{1}{2}(m\widehat{AB} + m\widehat{DC})$

D. $35° = \frac{1}{2}(m\widehat{AD} - m\widehat{BC})$

6. If *x* represents the measure of an inscribed angle of a circle and *y* represents the measure of the arc intercepted by the inscribed angle, then which of the following statements is true?

A. x = y

B. x = y/2

C. y = x/2

D. x = 2y

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