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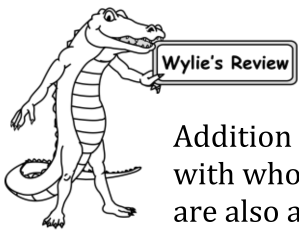
Table of Contents with Standards

Add and Subtract Integers

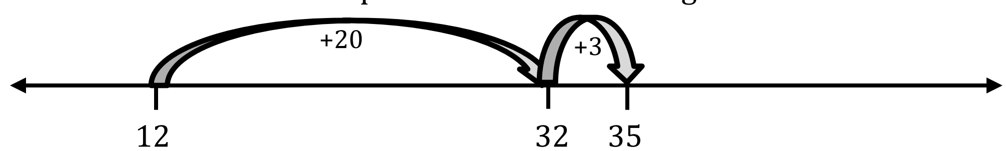
Extend previous understanding of operations with integers.

Apply and extend previous understandings of operations with whole numbers to add and subtract integers with procedural fluency.

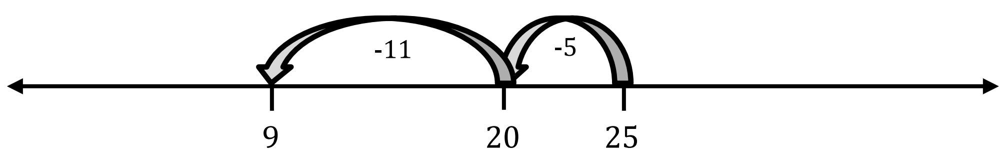


Addition and subtraction are mathematical operations that have been used with whole numbers, positive rational numbers, and absolute values. Integers are also added and subtracted. Integers are positive or negative in value. Positive values are located to the right of zero on a number line and negative values are located to the left of zero on a number line. When adding and subtracting whole numbers, the plus and minus signs determine which direction that the sum or difference will be from the first addend or the minuend.

The addition of 12 and 23 can be represented on a number line as shown. The first addend is marked on the number line and the second addend is decomposed and added in chunks to the first which has a sum greater than the first addend because a positive number is being added. 12+23

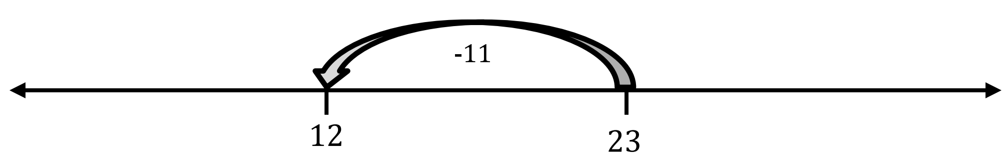


The subtraction of 25 and 9 can be represented on a number line as well. Both values are placed on the number line and the difference between the values is determined by counting up from the lesser value or counting back from the greater value. $25 - 9$



The operations do not change when adding and subtracting integers. The integers change the starting and ending points for the operations. When 23 was added to -11, the process would look as shown on a number line.

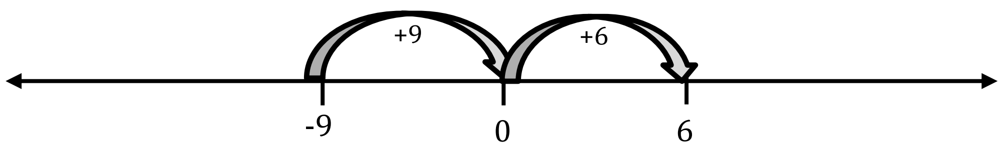
$$23 + (-11)$$



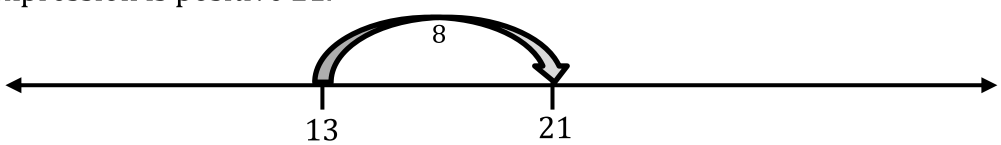
The starting point is 23 and -11 is being added. The number line is a model that demonstrates starting at 23 and adding negative 11. The negative sign means that the number being add is the opposite of positive 11 and the number being added will move to the left of the first value.

Example 1: What is the value of the expression $-9 + 15$?

-9 placed on the number line and +15 is added. Starting at the negative value the positive value is added. The positive number moves the value of the expression to the right of -9. As shown on the number line, 9 of the 15 takes the value of the expression to 0 and the additional 6 brings the value of the expression to positive 6.



Adding a negative, is modeled on a number line using the same process as subtracting a positive value. A negative value can be subtracted from a positive or negative number. When subtracting a negative number, the subtraction sign is thought of as the "opposite". The expression $13 - (-8)$ would be the same as 13 adding the opposite if -8. The value of the expression is positive 21.

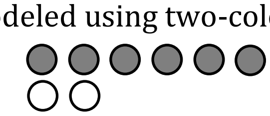


If the expression was a negative minus a negative, the operational process would be completed. With the expression $-16 - (-7)$, this is the same as -16 adding the opposite of -7. To find an equivalent expression, start at -16 and add the opposite of -7. The value of the expression is -9.

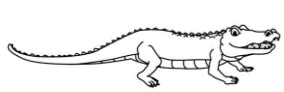
Example 2: Evaluate the value of the expression $41 - (-12)$.

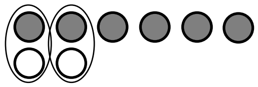
The first addend is 41 and the opposite of -12 is added to it for a total of 53.

The value of the expressions can be modeled using two-color counters. The expression $6 + (-2)$ is shown below.



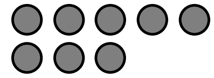
The top row represents the first added and the second row represents the second addend. If this were a number line, the second value would be moving to the left of the or the first addend. In this model, the counters are lined-up to show that the first two grey counters are even with the two white counters. The counters model "canceling out".



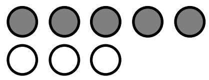


The grey counters that are leftover are the value of the expressions.

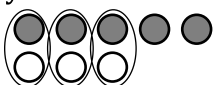
This can be modeled with negative values as well. $-5-(-3)$



The negative value is subtracted. This means that the opposite of -3 is added to -5. This is shown in the model by flipping-over the three grey counters to be the opposite.



The grey and white counters create values that are cancelled or zeroed out.



The value of the expression is -2.

Subtraction is the inverse operation of addition. Think of q and p that represent integers, these integers can be written as equations where $p-q=p+(-q)$ and $p+q=p-(-q)$, therefore, the expression $6+(-2)$ is equal to $6-2$ and the expression $41-(-12)$ is equal to $41+12$.

Example 3: Evaluate the following expression $-6-(-9)$.

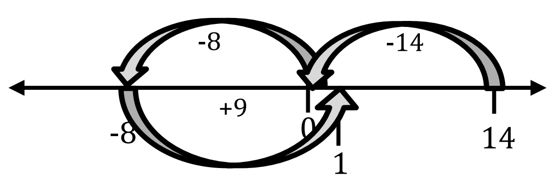
The expression is negative 6 minus negative 9. This is the same as -6 plus the opposite of negative 9 or 6 added to 9. This is shown rewritten as 9 minus 6. The expression has a value of 3.

$$-6-(-9)$$
$$-6+9$$
$$9-6$$
$$3$$

The expressions can have more than two integers being added and/or subtracted. $-12+15-(-8)$ Properties of operations can be used to arrange the numbers in a different order for adding. If it is easier to add the values different order, than the values may be moved. $15+(-12)-(-8)$ By moving the values, negative 12 can be compared to positive 15 and a positive 3 is the sum. The expression is now $3-(-8)$ or $3+8$, giving the expression a value of 11.

Example 4: Find the value of the expression $14-22-(-9)$.

In this expression, has three terms. The number line represents the adding of the values. First, 14 is marked on the number line. Then, -22 is added to 14, or it could be said that 22 is subtracted from 14. Lastly, the opposite of -9 is added. The solution is a positive 1.



Adding and subtracting integers is a real-life application. The table below shows how these expressions would be viewed in the real-life.

Expression	**Real-World Example**
$28 + 12$	Darnell has 28 dollars and earns 12 dollars more.
$28 - 12$	Darnell has 28 dollars and spends 12 dollars on a hat.
$28 + (-12)$	Darnell has 28 dollars and owes his brother 12 dollars.
$-28 + 12$	Darnell is short 28 dollars to buy a gaming system, and he is given 12 dollars.
$-28 + (-12)$	Darnell owes his parents 28 dollars and owes his brother 12 dollars.
$-28 - (-12)$	Darnell owes his parents 28 dollars and he earned 12 dollars which he paid towards his debt.

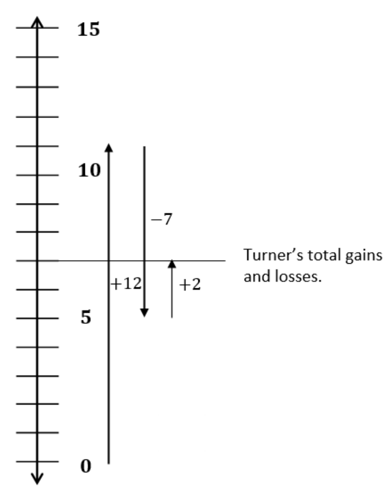
This table uses the same numbers in opposite forms to showcase what the numbers represent in real-life. These are not the only scenarios but are examples.

Example 4: Coach Carson recorded the gains and losses of the last three plays that his running back, Turner, ran. In the first play, Turner gained 12 yards, in the second play he lost 7 yards and in the third play he gained 2 yards. What is the total amount of yards that Turner gained or lost?

The model shows that Turner stated by gaining 12 yards. $(+12)$
He then lost 7 yards. (-7)
He then gained back 2 of the yards that he lost. (-2)

This expression represents the real-life scenario is $12 + (-7) - (-2)$.

The expression is equivalent $+7$. Turner gained a total of 7 yards on the three plays.



Now Try These:

For 1- 18, Equation Editor

1. What is the value of the expression $2 + (-7)$?

2. What is the value of expression $-6 + 13$?

3. What is the value of the expression $-20 - 15$?

4. Evaluate the expression $24 - (-16)$.

5. What is the value of expression $-34 - (-26)$?

6. Evaluate the expression $19 + (-5) - 32$.

7. Evaluate the expression $64 - (-21)$.

8. What is the value of the expression $-37 + (-18) + 6$?

9. What is the value of the expression $-10 + 26 - (-3)$?

10. What is the value of the expression $40 - 17 + (-22)$?

11. Evaluate the expression $-52 + 47$.

12. Evaluate the expression $2 + (-10) - (-5)$.

13. What is the value of the expression $-16 - 14$?

14. Evaluate the expression $100 - (23) + 18$.

15. The temperature was 76°F. When the sunset, the temperature decreased 29°F. What is the current temperature?

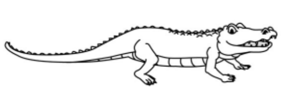
16. Cecelia weighs herself once a month during wrestling season. The differences in her weight each month is shown below.

Month	**Weight (gain/ loss)**
September	$+6$
October	-4
November	-1

What is the total amount of change in Cecelia's weight, in pounds?

17. Tyrek bought a basketball hoop for his driveway. He paid \$299 from his account. He now has \$176 in his account. How much money did Tyrek have in his account before he bought the basketball hoop?

18. The local produce stand bought 34 pounds of strawberries on Monday. By Friday, the stand sold 27 pounds of strawberries and received an order of 24 pounds of strawberries Saturday morning. How many pounds of strawberries does the produce stand have to sell on Saturday?



Problem-Solving with Percentages

Understand ratio and unit rate concepts and use them solve problems.

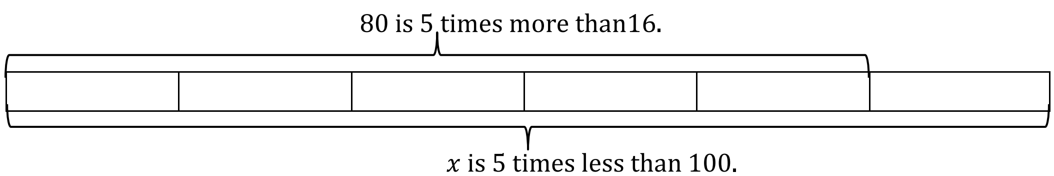
Apply ratio relationships to solve mathematical and rea-world problems involving percentages using the relationship between two quantities.

When a ratio compares a number to 100, the comparison can be written as a percent. Percentages can also be thought of as rates per hundred**.** Any given percent can be written fractionally as $\frac{x}{100}$. Therefore, hundredths and percentages are equivalent, although the notation is different. 20% is the same as $\frac{20}{100}$, 20:100, and 20 per every 100. Percentages determine a part of a whole when compared to a part per hundred. American coins could be thought of as a percent of one dollar or 100 cents. A nickel is five cents, it signifies 5 cents out of the 100 cents of one dollar or $\frac{5}{100}$ of one dollar. A nickel is 5% of the value of one dollar or 5% of 100 cents.

Example 1: Hector scored an 80% on his math test. He had 16 questions correct. How many questions were on Hector's test?

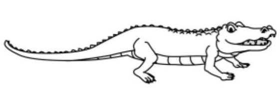
Hector scored 80% or $\frac{80}{100}$ of the points available and he had 16 questions correct. A bar diagram could be used to determine the number of questions on the assessment or 80% of ? $=$ 16 or $\frac{16}{x}$ has the same common ratio as $\frac{80}{100}$.

80 is 5 times greater than 16. The bar model shows that to find the number of questions on the assessment it will be 5 times less than 100.



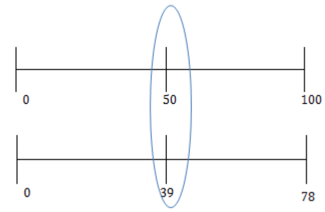
The number of questions on Hector's test is 20 because 20 is 5 times less than 100. Hector had 16 out of 20 questions correct.

Percentages are used to find the part when given the percent, by understanding that the whole is being divided into 100 parts and then selecting a part of the whole (the percent). A double number line illustrates one way to solve this type of problem.



Example 2: What is 50% of 78?

Create a number line to represent the first value, 50% or $\frac{50}{100}$.
Create another number line directly under the first to represent the other value in your ratio, 78.

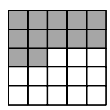


50 is the halfway point between 0 and 100. Then fill in equivalent ratios to locate the solution. Find the halfway point on the bottom number line to align with 50% on the top number line. $\frac{1}{2}$ of 78 $=$ 39

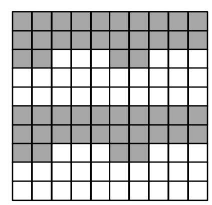
Therefore, 50% of 78 is 39.

Just as ratio tables we used to show equivalent ratios they can be used to organize information to find percentages of a number or to showcase the relationship between a number and its part per 100.

Example 3: There are 25 squares. 12 of them are shaded. What percent of the grid is shaded?



A percentage can be determined by looking at a part per 100. The grid shown has 25 squares, but what is it had 100 squares. How many of the squares would be shaded?



The model shows that 100 is 4 times larger than 25. The original 25 had 12 shaded. When the model is adjusted to 100, there are 48 total shaded squares. This is 48 parts out of the whole 100 or 48%. This ratio is the same as 12 shaded parts of the whole when the whole is 25. When 12 of the 25 parts are shaded, there is 48% of the grid shaded.

Example 4: Cassandra is buying a dress from a rack that has a 40% off sign on it. The dress has a cost of $89. How much of a discount is Cassandra receiving on her dress?

A ratio table can be used to calculate the discount on the dress.

Percentage of the Cost	100%	10%	20%	40%
Amount of Money	$89	$8.90	$17.80	$35.60

The first column defines 100% of the cost as 89.00. 10% of the cost of the dress can be calculated in multiple ways, thinking about what $\frac{1}{10}$ of the whole is can be used to calculate 10% of the whole. 20% is double of 10% and 40% is double of 20%. Cassandra will save $35.60 of the original cost when buying the dress.

Now Try These:

For 1-20, Equation Response:

1. There are 292 sixth graders at Beachland K-8 school. 75% of the sixth graders buy school lunch. How many sixth graders buy school lunch?

2. Jeremiah is a forward for his basketball team. In his best game this year, he scored 27 of the teams' 72 points. What percentage of the points did Jeremiah play?

3. At the Sassy Bakery, 35% of the cupcakes they sell are vanilla cupcakes with chocolate frosting. If the bakery sells 220 cupcakes, how many should be vanilla with chocolate frosting?

4. A number of students were surveyed about the type of pets they have. There were 10 students or 8% of the students surveyed had reptiles as pets. How many students were surveyed?

5. Mrs. Walker would like to walk 10,000 steps every day. She has walked 6,300 steps so far today. What percentage of her daily steps has she walked today?

6. Francesca earned $8,720 working last year at the grocery store. Her goal was to save 15% of the money she earned. How much money should Francesca have saved?

7. John is watching his sugar. He was consuming 115 grams of sugar a day. He would like to cut his sugar to 92 grams of sugar a day. What percent is John cutting his sugar back?

8. Lashaun ran 48 miles to train for cross country running. This week he ran 25% less because it was raining. How many miles less did Lashaun run this week?

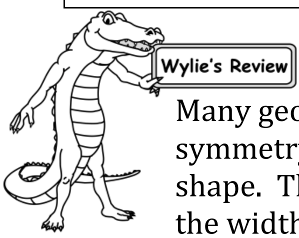
9. Patricia drank 48 ounces of water at this point today. This is 50% of her goal. How many ounces of water does Patricia intend on drinking today?

10. Hunter put $500 dollar into an account that will earn 2% interest in 6 months. How much interest will Hunter earn on his money after 6 months?

Area of Right Triangles

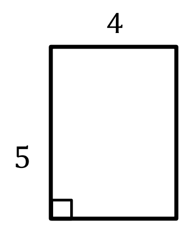
Model and solve problems involving two-dimensional figures and three-dimensional figures.

Derive a formula for the area of a right triangle using a rectangle. Apply a formula to find the area of a triangle.



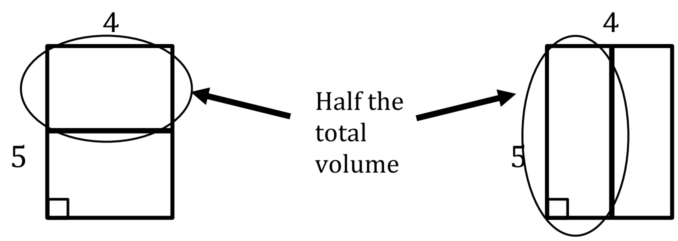
Many geometric shapes have at least one line of symmetry. The line of symmetry creates two equal parts or congruent halves of that particular shape. The area of a rectangle or a square can be determined by multiplying the width (base) of a side by the length (height.) These concepts of finding the area of a rectangle are to be applied to finding the area of triangles and other polygons by transforming the shapes into rectangles.

Think about a rectangle. The rectangle that has a width of 4 and length of 5 has an area of 20 units².

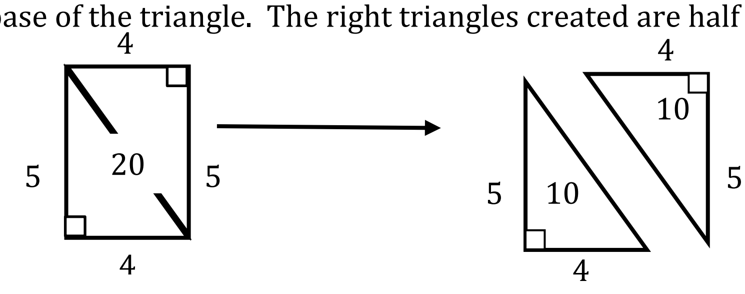


$A = b \times h$
$A = 4 \times 5$
$A = 20 units^2$

The rectangle has several lines that would cut the shape into two equal sized pieces. If the line of symmetry were placed to cut the rectangle horizontally or vertically in half then the area of the rectangle would be half of the total area of the rectangle.

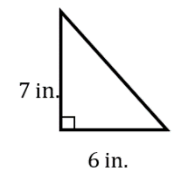


Use a diagonal line to decompose the rectangle into right triangles. The area of the rectangle is decomposed into two equal sections and the area of a single triangle is half the area of the total area of the rectangle. The height of the rectangle is the height or altitude of the triangle and the width of the rectangle is the base of the triangle. The right triangles created are half the area.

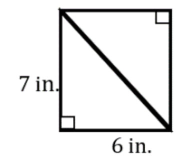


$A = 20\, units^2$
$A = 20 \div 2$
$A = 10\, units^2$

Example 1: Find the area of a right triangle (⊿).



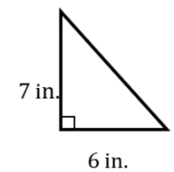
The right triangle represents half of a rectangle. The rectangle is completed by creating a second triangle the same shape and size and reflecting it.



Find the area of the completed rectangle.

$$A = b \times h$$
$$A = 6 \times 7$$
$$A = 42\ in^2$$

The rectangle will be decomposed into two congruent right triangles. The area of one right triangle is equivalent to half the total area of the rectangle.



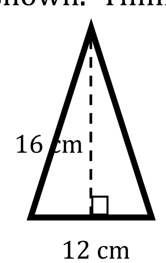
$$42 \div 2 = 21\ in^2$$

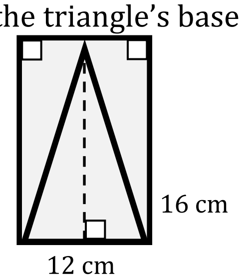
The area of the triangle is half the area of the rectangle. The area of this right triangle is 21 in^2.

The area of the right triangle was shown as being half the area of a rectangle where the right triangle has sides which are the base and height of the rectangle. The method for finding the area of the right triangle decomposes a rectangle into two halves, however not all triangles are right triangles. The area of these triangles can be determined by composing rectangles.

Example 2: Find the area of the triangle.

Look at the triangle shown. Think about the triangle's base being the base of a rectangle.

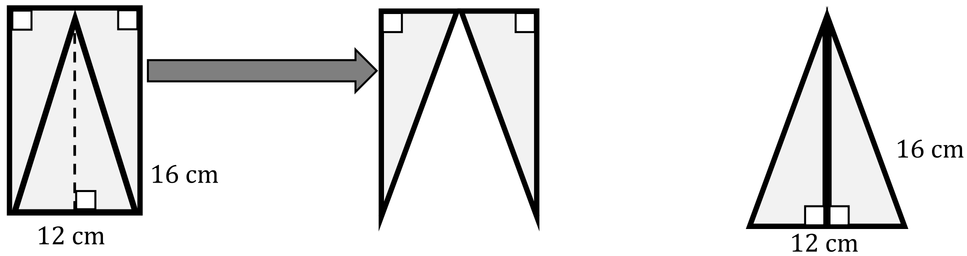




The triangle shares the base length with the rectangle. The rectangle will have the same base and in this case the height, or altitude, as the triangle. Calculate the area of the rectangle.

$$Area = base \times height$$
$$Area = 16 \times 12$$
$$Area = 192\ cm2$$

The triangle in this example is not created by cutting the rectangle in half diagonally, however, the rectangle can be decomposed to show two triangles that can be put together to create a congruent triangle.



The two triangles that are used to create the rectangle around the triangle.

The original triangle with the same measurements as the two triangles put together.

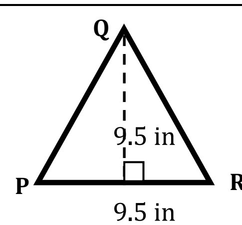
Since there are two congruent triangles completing the rectangle, the area of the triangle is half the area of the rectangle. The area of the rectangle is divided by two or multiplied by one-half. Area of rectangle ÷ 2 = Area of the triangle or a formula of $A = bh \div 2$ or $A = \frac{bh}{2}$

$$192 \div 2 = Area\ of\ triangle$$

$$96\ cm^2 = Area\ of\ triangle$$

As seen in these examples, the area of a triangle can be calculated by composing a rectangle with the same base and height measurements as the triangle. This will work for any triangle. Therefore, the area of a triangle can be calculated by using the formula for the area of the rectangle and multiplying by half. The formula would be $Area = \frac{1}{2}bh$.

Example 3: Find the area of the ΔPQR.



Find the area of a triangle using the formula.

$$Area\ of\ a\ \Delta PQR = \frac{1}{2}\ bh$$

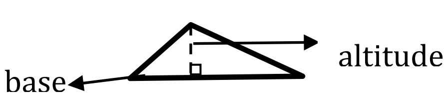
$$Area\ of\ a\ \Delta PQR = \frac{1}{2}(9.5 \times 9.5)$$

$$Area\ of\ a\ \Delta PQR = \frac{1}{2}(90.25)$$

$$Area\ of\ a\ \Delta PQR = 45.125\ in^2$$

This can be proven by composing a rectangle around the triangle finding the area of the rectangle and decomposing it back into 2 congruent triangles. Each of the congruent triangles have an area of exactly half of the composed rectangle. This proves the formula of $Area = \frac{1}{2}bh$.

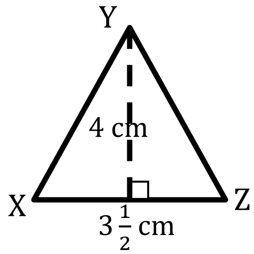
The base can be any side of the triangle and the height, or altitude, of the triangle is a line from the center of a vertex opposite of the base to the base which forms a right angle.



Now Try These:

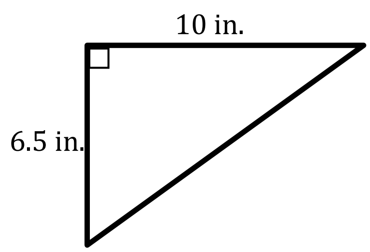
For 1-10, Equation Editor

1. Triangle XYZ is shown.



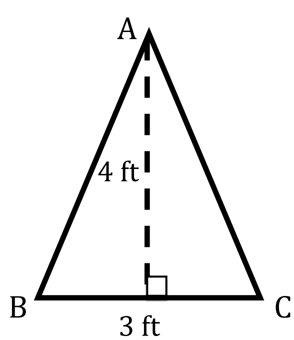
What is the area, in square, centimeters, of $\triangle XYZ$?

2. Find the area of the triangle below.



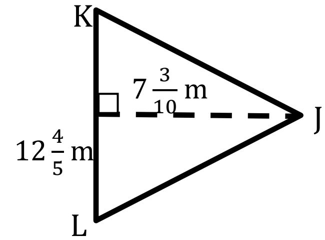
What is the area, in square inches, of the right triangle?

3. Triangle ABC is shown.



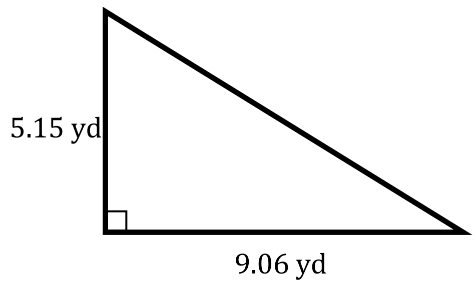
What is the area, in square feet, of triangle ABC?

4. Triangle JKL is shown.



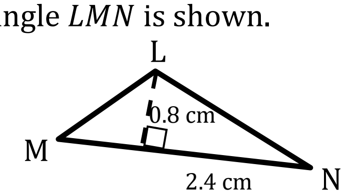
What is the area, in square meters, of $\triangle JKL$?

5. A triangle is shown.



What is the area, in square yards, of the triangle?

6. Triangle LMN is shown.



What is the area, in square centimeters, of $\triangle LMN$?