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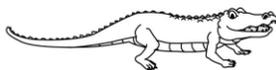


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**Everglades K-12 Publishing's Mathematics B.E.S.T. Standards Algebra 1
Algebraic Reasoning - MA.912.AR.2.1**

Write and Solve Linear Equations

MA.912.AR.2: Write, solve and graph linear equations, functions and inequalities in one and two variables.

MA.912.AR.2.1: Given a real-world context, write and solve one-variable multi-step linear equations.

REVIEW: An **algebraic expression** is a mathematical phrase containing one or more variables such as $a + b$, $3x + 2$.

- ❖ An **equation** is a mathematical sentence that uses an equal sign. An equation is true if the expressions on both sides of the equal sign are equal. The equation is false if they are not equal. Examples of equations are $6 - 2y = 12$ and $4 + n = 7$.
- ❖ A **numerical expression** is a mathematical phrase containing numbers only and no variables such as $3 + 5$.
- ❖ A **variable** is a symbol, usually a letter, which represents a value of a quantity which varies. Some examples of variables are x , y , n , μ .

Identifying Key Words:

Addition		Subtraction	Multiplication	Division	Equal To
Sum	More	Difference	Product	Quotient	Is
Plus		Minus	Times	Ratio	Has a result of
And		Reduce	Of	Average	Equals
Total		Decrease	Twice	Out of	
Increase		Less			

Tips for creating equations for real world applications:

- Assign a variable to represent the unknown value. Write down what the variable represents. If possible, express any other unknowns in terms of the variable.
- Write an equation using the variable and the relationship in the problem.
- Solve the equation.
- Answer the question in the problem. Check the answer.

Example 1: Write the following sentences as equations.

- a. The product of 9 and a number b is 54.
- b. The sum of a number m and 7.1 has a result of 20.5.
- c. The quotient of 200 and a number n equals ten.
- d. Twelve is equal to the difference of a number x and 5.

Solution:

- | | |
|-------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| <ol style="list-style-type: none"> a. $9b = 54$ b. $m + 7.1 = 20.5$ | <ol style="list-style-type: none"> c. $200 \div n = 10$ d. $12 = x - 5$ |
|-------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|



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Algebraic Reasoning - MA.912.AR.2.1

Example 2: Use words to describe each algebraic equation.

a. $6c = 36$

b. $3x + 1 = 17$

Solution:

a. The product of 6 and a number c is 36.

b. The sum of three times a number x and 1 is equal to 17.

Example 3: Solve the equation $5x - 13 = 3x + 7$.

Solution:

$$5x - 13 = 3x + 7$$

$$5x - 13 - 3x = 3x + 7 - 3x \quad \text{Subtract } 3x \text{ from both sides.}$$

$$2x - 13 = 7$$

$$2x - 13 + 13 = 7 + 13 \quad \text{Add 13 to both sides.}$$

$$\frac{2x}{2} = \frac{20}{2}$$

$$\frac{2}{2} = \frac{20}{2}$$

$$x = 10$$

Example 4: Jean made grades of 94, 68, and 88 on her first three tests in Civics. If the final counts double, what is the minimum grade Jean must make on her final to average an 80?

Solution: Let x denote the grade of her final. The average of three tests and a double final is

$$\frac{94 + 68 + 88 + x + x}{5} = 80$$

$$250 + 2x = 400$$

Simplify the numerator and multiply both sides by 5.

$$250 + 2x - 250 = 400 - 250$$

Subtract 250 from both sides.

$$\frac{2x}{2} = \frac{150}{2}$$

Divide by 2.

$$x = 75$$

Jean needs a minimum grade of 75 on her final.

Example 5: Find two consecutive odd numbers such that twice the second plus the first is 121.

Solution: Denote the two consecutive odd numbers by x and $x + 2$. Then

$$2(x + 2) + x = 121$$

$$2x + 4 + x = 121 \quad \text{Distribute.}$$

$$3x + 4 = 121 \quad \text{Combine like terms.}$$

$$3x + 4 - 4 = 121 - 4 \quad \text{Subtract 4 from both sides.}$$

$$\frac{3x}{3} = \frac{117}{3} \quad \text{Divide by 3.}$$

$$x = 39 \quad \text{The first odd integer is 39 and second odd integer is 41.}$$



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Algebraic Reasoning - MA.912.AR.2.1

Example 6: The perimeter of a rectangle is 44 cm. The length is twelve centimeters more than the width. Find the length and width of the rectangle.

Solution: Denote the width of the rectangle by w . Then the length of the rectangle is $l = 12 + w$. Since the perimeter is 44 cm, use the perimeter of a rectangle formula $p = 2w + 2l$.

$$\begin{array}{ll}
 p = 2w + 2l & \\
 44 = 2w + 2(12 + w) & \text{Substitute} \\
 44 = 2w + 24 + 2w & \text{Distribute.} \\
 44 = 4w + 24 & \text{Combine like terms.} \\
 44 - 24 = 4w + 24 - 24 & \text{Subtract 24 from both sides.} \\
 \frac{20}{4} = \frac{4w}{4} & \text{Divide by 4.} \\
 5 = w &
 \end{array}$$

The width of the rectangle is 5 cm and the length is $12 + w = 12 + 5 = 17$ cm.

Example 7: Susan earns \$120 for working an 8 hour shift. How much money does she earn per hour?

- A. \$5 B. \$10 C. 1\$5 D. \$20 E. \$25

Solution: Denote Susan's hourly rate by x . The equation is $8x = 120$. Divide both sides by 8 to obtain $x = 15$. The correct answer is C.

Example 8: Hummingbird food should be a 25% sugar solution. Gregory has pure water and a solution that is 60% sugar. How much of the 60% sugar solution should he combine with the water to make 4 cups of hummingbird food for the feeder?

Solution: Let x be the number of cups of 60% solution to be included in the mixture.

Organize your information:

$$\begin{array}{l}
 0.6x + 0(4 - x) = 0.25(4) \\
 0.6x = 1.00 \\
 x = 1\frac{2}{3}
 \end{array}$$

Solution	% Sugar	Cups
60%	0.6	x
Water	0	$4 - x$
25%	0.25	4

Gregory needs $1\frac{2}{3}$ cups of the 60% sugar solution.



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Algebraic Reasoning - MA.912.AR.2.1

Now Try These:

1. **Editing Task Choice:** Chose the equation from the box to replace the highlighted text that makes the statement below true.

The sum of an integer x and the preceding integer is 51 can be represented as $2x + 1 = 51$.

$$2x + 1 = 51$$

$$2x - 1 = 51$$

$$2x + 2 = 51$$

$$2x - 2 = 51$$

For 2-3, Equation Editor: Write each sentence as an equation.

- Eight is the difference between five times x and seven.
- The product of an even number x and the next consecutive even number is 24.
- The quantity of three times y plus two multiplied by the quantity of y minus three is 100.

For 4-16, Open Response: Create equations and then solve the following problems.

4. A rectangular piece of a land with length a meters and width 80 m less than its length has perimeter of 240 m. Find the length and width of this land.

5. The first shelf has x books on it, the second shelf has twice as many books as the first shelf and the number of books on the third shelf is the first and second shelf combined. There are a total of 72 books. How many books are on each shelf?

6. A father is twice as old as his son and the sum of their ages is 75. How old is the father?

7. There are three more apple trees than pear trees in the yard. Altogether there are 35 trees. How many apple trees and how many pear trees are there?

8. There are 32 students in the class. Fourteen of them received a "C" on their Algebra test, the rest of them received "A"s and "B"s. There were twice as many "B"s than "A"s. How many "A"s, "B"s, and "C"s were there?

9. Gregg subtracted some number x from 47, then multiplied the result by two, subtracted 15 and got 45. What was Gregg's number?

10. Think of a number, divide it by 5 and then add 27 to the result. The resulting number is half of the original number. What was the original number?



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Everglades K-12 Publishing's Mathematics B.E.S.T. Standards Algebra 1
Algebraic Reasoning - MA.912.AR.3.1

Problem Solving over Real Number Systems

MA.912.AR.3: Write, solve, and graph quadratic equations, functions and inequalities in one and two variables.

MA.912.AR.3.1: Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.

REVIEW: A **quadratic equation** is an equation that can be written as

$$ax^2 + bx + c = 0, \text{ where } a \neq 0.$$

Quadratic equations can be solved by taking square roots, factoring, completing the square, and using the quadratic formula. The **solutions** of a quadratic equation are the values of the variable x that make the equation a true statement. The maximum number of solutions is two, but there can also be just one solution (double solution), or no real solutions. The solutions may be verified by substituting them back into the original equation and making sure that they work.

Some quadratic equations are easy enough to be solved by inspection and taking the square roots.

Example 1: Solve for x : $x^2 - 4 = 0$.

Solution: Since $x^2 - 4 = 0$, we have $x^2 = 4$, and hence $x = \pm\sqrt{4} = \pm 2$. Thus, the quadratic equation $x^2 - 4 = 0$ has two solutions: $x = 2$ and $x = -2$.

Example 2: Solve for x : $3x^2 + 27 = 0$

Solution: First, divide both sides of the equation by 3 to get $x^2 + 9 = 0$. Subtracting 9 from both sides gives $x^2 = -9$. Since a square of a number cannot be negative, this equation has no real solutions.

Some quadratic equations allow you to factor out a common term and then use the Zero-Product Property. This property says that if a product $p \cdot r = 0$, then either $p = 0$ or $r = 0$.

Example 3: Solve $3x^2 + 2x = 0$.

Solution: Factor out x to get $x(3x + 2) = 0$. According to the Zero-Product Property, when the product of factors is zero, then one or more of its factors must be zero. Hence, either $x = 0$ or $3x + 2 = 0$. Solving both equations yields two solutions $x = 0$ and $x = -\frac{2}{3}$.



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Algebraic Reasoning - MA.912.AR.3.1

If the quadratic equation $x^2 + px + q = 0$ can be **factored** as $(x + a)(x + b) = 0$ for some real numbers a and b , where $a + b = p$ and $ab = q$, then the Zero-Product Property states that either $x + a = 0$ or $x + b = 0$. Thus, the two solutions are $x = -a$ and $x = -b$.

Example 4: Use factoring to solve $x^2 + 5x + 6 = 0$.

Solution: Here, $p = 5$ and $q = 6$. Hence, we need to find two numbers a and b such that $a + b = 5$ and $ab = 6$. Taking $a = 3$ and $b = 2$ we get $x^2 + 5x + 6 = (x + 3)(x + 2) = 0$. From here, either $x + 3 = 0$ or $x + 2 = 0$ and thus the solutions are $x = -3$ and $x = -2$.

Completing the square makes use of the algebraic identity: $x^2 + 2bx + b^2 = (x + b)^2$.

To complete the square for $x^2 + kx$ or $x^2 - kx$, add $\left(\frac{k}{2}\right)^2$; that is, add the square of half the coefficient of x .

$$(1) \quad x^2 + kx + \left(\frac{k}{2}\right)^2 = \left(x + \frac{k}{2}\right)^2$$

$$(2) \quad x^2 - kx + \left(\frac{k}{2}\right)^2 = \left(x - \frac{k}{2}\right)^2$$

Example 5: Solve the equation $x^2 - 2x - 3 = 0$ by completing the square.

Solution: Rewrite the equation to complete the square on the left-hand side:

$$x^2 - 2x = 3$$

Move 3 to the right side of the equation.

$$x^2 - 2x + 1 = 3 + 1$$

Add 1 (which is one-half of the coefficient 2 of x) to both sides to complete the square on the left.

$$(x - 1)^2 = 4$$

Complete the square.

Now, either $x - 1 = 2$ (hence $x = 3$) or $x - 1 = -2$ (hence $x = -1$).

Thus, the solutions are $x = 3$ and $x = -1$.

You should use the **quadratic formula** when factoring cannot be done.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic formula can be derived from the above completing the square technique.



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Algebraic Reasoning - MA.912.AR.3.1

Example 6: Use the quadratic formula above to solve the equation: $x^2 - 8x + 13 = 0$.

Solution: Here, $a = 1$, $b = -8$ and $c = 13$. Substituting the values of a , b and c into the quadratic formula gives:

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - (4)13}}{2}$$

$$x = 4 \pm \frac{\sqrt{4}}{2} \sqrt{16 - 13}$$

$$x = 4 \pm \sqrt{3}$$

Hence, the two solutions are $x = 4 + \sqrt{3}$ and $x = 4 - \sqrt{3}$.

In the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

the quantity $b^2 - 4ac$ is called the **discriminant**, because it can "discriminate" between the possible number of real solutions:

- when $b^2 - 4ac > 0$ (positive), you get two real solutions,
 - when $b^2 - 4ac = 0$, you get just one real solution (both answers are the same),
 - when $b^2 - 4ac < 0$ (negative) you get no real solutions (two non-real or complex solutions).
-

Example 7: Solve $2x^2 - 4x - 3 = 0$.

Solution: This quadratic equation cannot be factored and therefore we apply the quadratic formula. Here, $a = 2$, $b = -4$, and $c = -3$:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4} = \frac{4 \pm 2\sqrt{10}}{4} = \frac{2 \pm \sqrt{10}}{2}$$

Hence, the two solutions are $x = \frac{2 - \sqrt{10}}{2}$ and $x = \frac{2 + \sqrt{10}}{2}$.



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Algebraic Reasoning - MA.912.AR.3.1

Now Try These:

For 1-10, Equation Editor: Solve each quadratic equation by finding square roots. If the equation has no real-number solution, write *no solution*.

1. $b^2 - 121 = 0$

2. $-9x^2 + 9 = 0$

3. $n^2 + 121 = 0$

4. $4x^2 - 100 = 0$

5. $49 - 25u^2 = 0$

6. $2x^2 + 77 = 0$

7. $64w^2 = 16$

8. $\frac{1}{4}x^2 - 1 = 0$

9. $3m^2 - \frac{1}{12} = 0$

10. $7y^2 + 0.12 = 1.24$

For 11-18, Equation Editor: Solve each quadratic equation by using the zero-product property.

11. $y(y - 7) = 0$

12. $2x(x - 1) = 0$

13. $-2x + 5x^2 = 0$

14. $9x^2 - 4x = 0$

15. $12x - 3x^2 = 0$

16. $-3x^2 + 6x = 0$

17. $x + 3x^2 = 0$

18. $x^2 - 10x = 0$

For 19-28, Equation Editor: Solve each quadratic equation by factoring.

19. $x^2 + 3x - 4 = 0$

20. $m^2 - 7m + 12 = 0$

21. $x^2 + 3x - 28 = 0$

22. $x^2 - x - 12 = 0$

23. $m^2 - 5m - 24 = 0$

24. $a^2 + 4a = 21$

25. $x^2 + 15 = 8x$

26. $2x^2 - x - 21 = 0$

27. $5x^2 + 27x - 18 = 0$

28. $3q^2 + q = 14$



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Everglades K-12 Publishing's Mathematics B.E.S.T. Standards Algebra 1
Functions - MA.912.F.1.5

Comparing Linear Functions

MA.912.F.1: Understand, compare and analyze properties of functions.

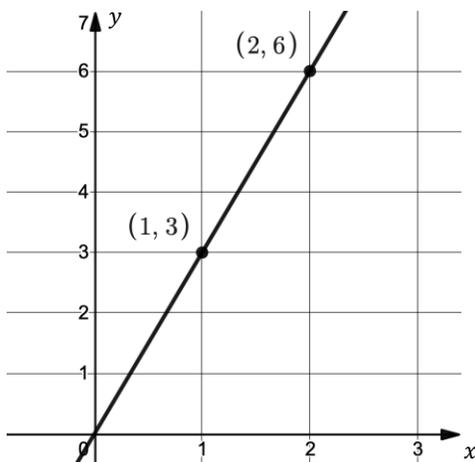
MA.912.F.1.5: Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.

REVIEW: Functions can be represented algebraically, graphically, numerically in tables, or by verbal descriptions. In this section we work with linear functions only.

Any **linear function** $y = mx + b$ has a constant rate of change m . A **rate** describes how much one variable changes with respect to another. For example, rates are often used to describe relationships between time and distance. When an object or person moves at a constant rate, the relationship between distance and time is linear.

Example 1: Two functions are given below. Which one has the greater rate of change?

Function A



Function B

$$y = 2.5x$$

Solution: Function A seems to represent linear relationship, so we can pick any two points to calculate the rate of change. Let's pick points (1, 3) and (2, 6).

$$\begin{aligned} \text{rate of change}_A &= \frac{y_2 - y_1}{x_2 - x_1} = \\ &= \frac{6 - 3}{2 - 1} = \frac{3}{1} = 3 \end{aligned}$$

The rate of change of the Function B is given by its slope. Since the slope is 2.5, the rate of change is 2.5.

Hence, Function A has greater rate of change.

Example 2: Nickson and Jackson are competing on their bicycles to see who is faster. The distance covered by Nickson in every minute is given in the table below, while the distance covered by Jackson is given by the function $y = 0.15t$, where t is time measured in minutes. Who has greater speed?

Time (in minutes)	1	2	3	4
Distance (in miles)	0.2	0.4	0.6	0.8



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Solution: The speed or velocity is given by the rate of change. Hence, we need to find the rate of change for each. Nickson is covering equal distances during each time interval, hence he has constant speed (= constant rate of change). Pick two points to calculate his rate of change:

$$\text{rate of change}_{\text{Nickson}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.4 - 0.2}{2 - 1} = \frac{0.2}{1} = 0.2$$

Jackson's rate of change is the slope of his distance function: $m = 0.15$

Since Nickson's rate of change is larger than Jackson's, Nickson has greater speed.

Example 3: Tamara adds 5 new apps (applications) to her cell phone every month. She currently has 12 apps. Miguel has 23 apps, but is adding only 3 apps per month. How many months will it take to Tamara to have more apps than Miguel?

Solution: We will use the table to calculate the number of apps after each month.

Months	0	1	2	3	4	5	6	7
Tamara's apps	12	17	22	27	32	37	42	47
Miguel's apps	23	26	29	32	35	38	41	44

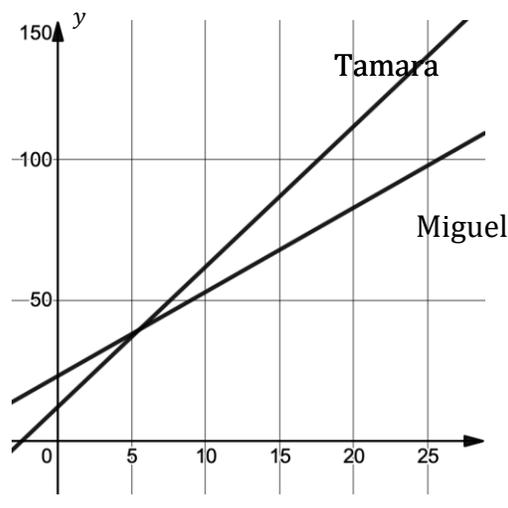
In the table, Tamara starts with 12 apps and adds 5 apps every month; Miguel starts with 23 apps and adds 3 apps every month. We see that after 6 months Tamara has more apps on her cell phone.

We can also come up with linear equations for each. Let t stand for months. Then Tamara's number of apps as a function of t is $f(t) = 12 + 5t$, and Miguel's number for apps as a function of t is given by $g(t) = 23 + 3t$.

Graphically these linear functions are represented by two lines.

Since the line representing Tamara's apps has larger slope ($m = 5$), it is eventually (after 6 months) growing faster than the function representing Miguel's number of apps.

The y -intercepts in each case represent the original number of apps they each started with (12 and 23, respectively).



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Now Try These:

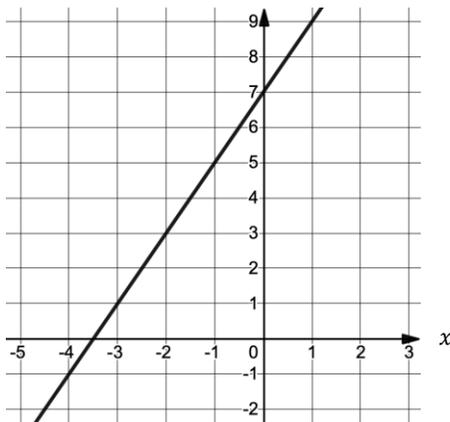
1. Equation Editor: Given two functions $y = 7 + 3x$ and $y = 8 - 10x$, which line has the greater

- a. slope? b. y-intercept?

For 2-4, Open Response: Compare the following functions to determine which has the greater rate of change. Explain.

2. Function 1: $y = 3x + 7$

Function 2:



3. Function 1: $y = 2x + 4$

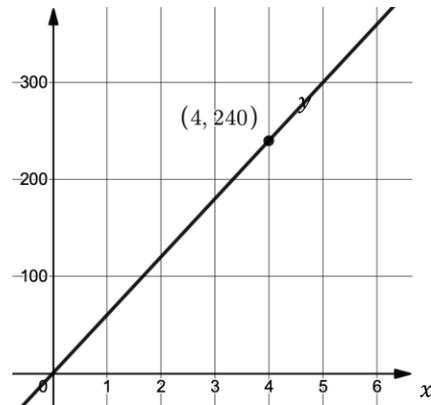
Function 2:

x	-1	0	2
y	-6	-3	3

4. Function 1: $y = 4x + 5$

Function 2: A function whose graph is a line and goes through the points (2, 6) and (3, 10).

5. Open Response: Erik drives his car and the distance travelled is shown on the graph to the right. Helina travels according to the rule $y = 50x$, where x is time in hours and y is distance in miles. Determine who has greater speed.



6. Open Response: Compare the two linear functions listed below and determine which has a negative slope.

Function 1: Samantha starts with \$20.00 on a gift card for the book store. She spends \$3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks.

Function 2: The school bookstore rents graphing calculators for \$5.00 per month. It also collects a non-refundable fee of \$10.00 for the school year. Let C be the total cost of renting a calculator as a function of the number of months m .



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Data Analysis and Probability - MA.912.DP.2.4

Modeling Bivariate Data

MA.912.DP.2 Solve problems involving univariate and bivariate numerical data.

MA.912.DP.2.4: Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.

REVIEW: The **Least Squares Regression Line** is a line in the form of $\hat{y} = a + bx$, where a represents the y-intercept and b represents the slope. The response in the model is denoted \hat{y} to indicate that these are predicted y values, not the true observed y values.

- ❖ The **independent variable** x is also called the explanatory or predictor variable.
- ❖ The **dependent variable** \hat{y} is considered as the response or outcome.
- ❖ Thus, the regression line is a straight line that describes how a response variable \hat{y} changes as an explanatory variable x changes.
- ❖ The **slope** b of the line indicates how much \hat{y} changes for a unit change in x .
- ❖ The **intercept** a is the value of \hat{y} for $x = 0$, that is, it is the point where the line crosses the y-axis. It may or not have a physical interpretation, depending on whether or not x can take values near 0.

Extrapolation - Extrapolation is the process of predicting a value outside the range of the data points.

Interpolation - Interpolation is the process of predicting a value within the range of the data points.

Example 1:

Philip tutors to make extra money for college. For each tutoring session, he charges a one-time fee of \$25.00 plus \$15.00 per hour of tutoring. A linear equation that expresses the total amount of money Philip earns for each session he tutors is $\hat{y} = 25 + 15x$.

- a. What are the independent and dependent variables?
- b. What is the y-intercept and what is the slope?
- c. Interpret the y-intercept and slope using complete sentences.



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Solution:

- a. The independent variable (x) is the number of hours Philip tutors each session. The dependent variable (y) is the amount, in dollars, Philip earns for each session.
- b. The y -intercept is 25 ($a = 25$). The slope is 15 ($b = 15$).
- c. The y -intercept represents the one-time initial charge that Philip charges at the beginning of the tutoring session (this is when $x = 0$). The slope \$15.00 represents how much his earnings are expected to increase for each additional hour that he tutors.

Example 2:

Data was collected to predict the length of the distance a golf ball will travel when hit by a golf club at a certain speed. The speed of the golf ball, s , is measured in miles per hour and the length of the distance the ball travels, \hat{y} , is measured in yards. The following formula gives the relationship $\hat{y} = 3.18 + 57.66(s)$. Interpret the slope and the y -intercept in this context. Comment on the reasonableness of the y -intercept.

Solution: The slope is $b = 57.66$ which tells us that for every 1 mph increase in the speed of the golf ball, the length of the distance traveled increases by approximately 57.66 yards. The y -intercept is $a = 3.18$ which means that if the ball is hit with a speed of 0 mph, then the model predicts that the ball travels 3.18 yards. The interpretation of the y -intercept doesn't make sense here because the speed of 0 mph means that the ball wasn't hit at all.

Example 3: The sales manager of a local pool supply store collected data on sales versus time for the last 5 years and found the points to lie approximately along a straight line. The data is given in the table where $t = 1$ corresponds to 2008.

Years, t	1	2	3	4	5
Sales in \$1000s	20	29	42	49	60

- a. By using the points corresponding to the first and fifth years, find an equation of the trend line.
- b. Is the slope of the line positive or negative? What does the sign of the slope tell you about the relationship between the sales and years?
- c. Interpret the slope and the y -intercept of this linear model.



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Solution:

- a. Using the points (1, 20) and (5, 60), we find that the slope of the line is given by $m = \frac{60-20}{5-1} = \frac{40}{4} = 10$. Using the point-slope form of the equation of a line with the point (1, 20) and slope $m = 10$, we obtain $y - 20 = 10(t - 1)$, that is, $\hat{y} = 10t + 10$.
- b. The slope is $b = 10$ and is positive. Positive relationship means that sales are increasing each additional year.
- c. The slope $b = 10$ means each additional year sales are increasing by \$10,000. The y -intercept $a = 10$ tells that at the beginning when $t = 0$, or in year 2007, the sales of the pool supply store were \$10,000.

Example 4: Rory's parents began measuring his height beginning at the age of two. Each year, on his birthday, they recorded his height as follows:

AGE	2	3	4	5	6	7	8	9	10
HEIGHT (in inches)	36	38	41	43	47	48	51	55	56

- a) Would it be reasonable to predict Rory's height at the age of 16?
- b) Would it be reasonable to predict Rory's height when he is 5 ½ years old?

Solution:

- a) It is not reasonable to predict Rory's height when he is 16. This is extrapolation.
- b) It is reasonable to predict his height at 5 ½. This is within the range of the x -values for this data.



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Data Analysis and Probability - MA.912.DP.2.4

Using technology to create the Least Squares Regression Line (LSRL). The TI-84 Graphing Calculator

When given all of the data points, you can use your calculator to find the LSRL.

Step 1: Go to STAT, and click EDIT. Then enter all of the data points into lists 1 and 2.

Step 2: Go to STAT, and click right to CALC. Then hit LinReg 8. Hitting enter will give you the slope and y-intercept of your LSRL.

L1	L2	L3	3
4	6		
7	7		
8	8		
9	9		
10	10		
10	10		
L3(1)=			

EDIT	TESTS
2:2-Var Stats	
3:Med-Med	
4:LinReg(ax+b)	
5:QuadReg	
6:CubicReg	
7:QuartReg	
8:LinReg(a+bx)	

LinReg
y=a+bx
a=5.432642487
b=.4961139896

How to Find the Least Squares Regression Line using *an* online application:

1. Input your data.
2. In the next line, under your table, type in $y_1 \sim mx_1 + b$. This will draw in the Least Squares Regression Line and give you the slope (b) and y-intercept (a).



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Data Analysis and Probability - MA.912.DP.2.4

Now Try These:

For 1-4, Equation Editor: Find the slope and y -intercept.

1. $y = 6 - 3x$
2. $2x - 5y = 7$
3. $8.6x = 4.3y + 12.9$
4. $18 - 6x + 2y = 0$

For 5-10, Multiple Choice: Determine the correct statement about the slope of the relationship between the two given variables.

5. Length of time spent studying and GPA.
 - A. It is positive
 - B. It is negative
 - C. It can be positive or negative
 - D. There is no relationship
6. Time surfing web and time outdoors.
 - A. It is positive
 - B. It is negative
 - C. It can be positive or negative
 - D. There is no relationship
7. Person's age and memory.
 - A. It is positive
 - B. It is negative
 - C. It can be positive or negative
 - D. There is no relationship
8. Number of friends and number of party invitations.
 - A. It is positive
 - B. It is negative
 - C. It can be positive or negative
 - D. There is no relationship
9. Grade in school and zodiac sign.
 - A. It is positive
 - B. It is negative
 - C. It can be positive or negative
 - D. There is no relationship
10. Number of crimes in a city and the number of patrolling officers.
 - A. It is positive
 - B. It is negative
 - C. It can be positive or negative
 - D. There is no relationship
11. **Open Response:** The equation $W = 3.51L - 192$ expresses the relationship between the length L (in feet) and the expected weight W (in tons) of adult blue whales. State and interpret the slope of this equation. Determine the y -intercept commenting on its meaningfulness.

